Problems

1. Each wealth-less agent has a project which requires an initial investment of £200. The project produces output valued at £500 if it succeeds and £0 when it fails.

   There are two types of agents. For type a agent, the project succeeds with probability 0.2 and fails with probability 0.8. For type b agent, the project succeeds with probability 0.8 and fails with probability 0.2.

   The lender lends to groups of two with a group lending contract as follows: Each agent in the group repays £300 when both her own and her peer's project succeed, £400 when her own project succeeds but her peer's project fails and £0 when her own project fails.

   (a) *Show that the type b prefers to group with type b as compared to type a.*

$$\begin{aligned}
EU_{i,j} &= \pi_i \pi_j \cdot \big[500 - 300\big] + \pi_i(1-\pi_j) \cdot \big[500 - 400\big] \\
&= \pi_i \pi_j \cdot \big[200\big] + \pi_i(1-\pi_j) \cdot \big[100\big] \\
EU_{b,b} &= \pi_b \pi_b \cdot \big[200\big] + \pi_b(1-\pi_b) \cdot \big[100\big] \\
&= 0.64 \cdot 200 + 0.16 \cdot 100 = 128 + 16 = 144 \\
EU_{a,a} &= \pi_a \pi_a \cdot \big[200\big] + \pi_a(1-\pi_a) \cdot \big[100\big] \\
&= 0.04 \cdot 200 + 0.16 \cdot 100 = 8 + 16 = 24 \\
EU_{a,b} &= \pi_a \pi_b \cdot \big[200\big] + \pi_a(1-\pi_b) \cdot \big[100\big] \\
&= 0.16 \cdot 200 + 0.04 \cdot 100 = 32 + 4 = 36 \\
EU_{b,a} &= \pi_b \pi_a \cdot \big[200\big] + \pi_b(1-\pi_a) \cdot \big[100\big] \\
&= 0.16 \cdot 200 + 0.64 \cdot 100 = 32 + 64 = 96
\end{aligned}$$

   It follows from these calculations that

$$EU_{b,b} > EU_{b,a}.$$

   (b) *Explain why type a is not able to group with type b even though she would like to.*

$$EU_{a,b} - EU_{a,a} < EU_{b,b} - EU_{b,a}$$

   LHS is the type a's benefit from grouping with type b as opposed to grouping with type a. RHS is type b's benefit from grouping with type b as opposed to type a. Thus, type a is not able to bribe type b to group with her.

2. Each borrower has a project which requires an investment of 1 unit of capital. With probability π^i the project succeeds and produces output x and with probability $1-\pi^i$, it fails and produces 0.

   When the agent exerts high effort, the project succeeds with probability $\pi^i = \pi^h$. Conversely, when the agent exerts low effort, the project succeeds with probability $\pi^i = \pi^l$

and the agents obtains a private benefit of value B. ($\pi^l < \pi^h$) The borrowers have no wealth and no alternative source of income and the lender's opportunity cost of capital is ρ. We assume that $x > \frac{\rho}{\pi^h} + \frac{B}{\pi^h - \pi^l}$ and that the lender has all the bargaining strength and extracts all the surplus from the borrower.

Let the borrower's payoff in individual lending be b_I if her project succeeds and 0 if it fails. Alternatively, the lender may lend to groups of 2. Let each borrower's payoff in group lending be b_G if both group members' projects succeed and 0 otherwise (if one or more member's project fails).

(a) *Write down the lender's problem in individual lending and group lending and find the optimal b_I and b_G.*

The lender's problem under individual lending is as follows.

$$\min_b \pi^h b_I$$
$$\text{subject to} \quad \pi^h b_I \geqslant 0 \qquad \text{(B-PC)}$$
$$\pi^h b_I \geqslant \pi^l b_I + B \qquad \text{(B-ICC)}$$
$$\pi^h (x - b_I) \geqslant \rho \qquad \text{(L-ZPC)}$$

Under the optimal contract (B-PC) would be slack, (B-ICC) would bind and (L-ZPC) would be slack under the conditions on x. Under an optimal contract, borrower's minimum payoff that would satisfy the (B-ICC) is given by $b_I = \frac{B}{\Delta\pi}$ and consequently the borrower's minimum expected rent is $E[b_I \mid h] = \frac{\pi^h}{\Delta\pi} B$.

The lender keeps all the surplus[1], $\pi^h \left(x - \frac{B}{\Delta\pi}\right) - \rho$, and the borrower just retains expected rents amounting to $\frac{\pi^h}{\Delta\pi} B$, which barely satisfies her incentive compatibility constraint.

The lender's problem under group lending is as follows.

$$\min_b \pi^h b_G$$
$$\text{subject to} \quad (\pi^h)^2 b_G \geqslant 0 \qquad \text{(G-PC)}$$
$$(\pi^h)^2 b_G \geqslant (\pi^l)^2 b_G + B \qquad \text{(G-ICC)}$$
$$\pi^h x - (\pi^h)^2 b_G \geqslant \rho \qquad \text{(L-ZPC)}$$

(G-PC) slack and (G-ICC) binds and (L-ZPC) [2] slacks under the conditions on x.

[1] per unit capital lent

[2] The group obtains output $2x$ with probability $(\pi^h)^2$ and produces x with the probability $2\pi^h(1-\pi^h)$. Since there is no ex post information problem the lender is able to obtain this output. In the event that both borrowers succeed the lender has to leave a minimum payoff of $2b_G$ to the borrowers in the group. This happens with probability $(\pi^h)^2$. Lender's zero profit condition is as follows.

$$(\pi^h)^2 \cdot 2x + 2\pi^h(1-\pi^h) \cdot x - (\pi^h)^2 b_G \geqslant 2\rho$$
$$\pi^h x - (\pi^h)^2 b_G \geqslant \rho \qquad \text{(L-ZPC)}$$

Minimum payoff that satisfies the (G-ICC) is given by $b_G \geqslant \frac{B}{(\pi^h+\pi^l)\Delta\pi}$. Consequently, minimum borrower's rent is given by $E[b_G \mid h,h] \geqslant (\pi^h)^2 \cdot \frac{B}{(\pi^h)^2-(\pi^l)^2}$.

The lender keep all the surplus[3], $\pi^h x - \frac{(\pi^h)^2 B}{(\pi^h)^2-(\pi^l)^2} - \rho$, and the borrower just retains minimum expected rents amounting to $(\pi^h)^2 \cdot \frac{B}{(\pi^h)^2-(\pi^l)^2}$ which satisfies her incentive compatibility constraint.

(b) *Show that the economic rents a borrower obtains in individual lending is higher as compared to the rent she would obtain in group lending.*

The borrower's rents are lower under group lending as compared to individual lending if the following condition holds:

$$E[b_G \mid h,h] < E[b_I \mid h]$$
$$(\pi^h)^{\not{2}} \cdot \frac{\not{B}}{(\pi^h+\pi^l)\not{\Delta\pi}} < \not{\pi^h}\frac{\not{B}}{\not{\Delta\pi}}$$

The condition always hold. Consequently, from the perspective of lending efficiency, the borrowers retain lower rent in the group lending and it is the more efficient of the two lending mechanism.

(c) *Find the least productive project financed in individual and group lending.*

A lending mechanism efficiency is judged by the economic rents absorbed by the information problem. In group lending, borrowers retain lower economic rents, which mean there is a greater surplus available. Given that in this case the lender retains all the surplus, it does seem like the borrower does not seem to be better off at first glance. But, it also implies that if lower bound of the projects financed by group lending is lower than individual lending. With individual lending, the lower bound is given by $x \geqslant \frac{\rho}{\pi^h} + \frac{B}{\Delta\pi}$. Conversely, with group lending, the lower bound is given by $x \geqslant \frac{\rho}{\pi^h} + \left(\frac{\pi^h}{\pi^h+\pi^l}\right)\frac{B}{\Delta\pi}$. The lower bound for projects feasible is lower in the group lending. It is interesting to note that $\frac{E[b_I|h]}{E[b_G|h]} = \frac{\pi^h}{\pi^h+\pi^l}$ and is gives us the proportion by which the rents in group lending are lower.

Discussion: The economic rents retained by the borrowers are actually wasteful. They create a wedge between the first best and the second best and lower the surplus of the project. The surplus of the project is defined by the expected output minus the expected payoffs and the opportunity cost of capital. The lending efficiency of a mechanism is judged by the economic rents retained by the borrower and lower the rents, the greater the efficiency. Thus, it implies that group lending is a more efficient lending mechanism than individual lending.

Lets try to understand this in the case where the lender has all the bargaining power and is able to push the borrower's payoff as much as he can. With group lending the borrowers retain lower rents and thus the lending efficiency is greater. This could result in the following outcomes: a) in this case since the lender has all the

[3] per unit capital lent

bargaining power, lender would like to lend to all the projects that give him non-negative returns, he would lender to lower productivity projects b) if the bargaining power would have been half and half, the surplus would have divided equally between the lender the borrower and the borrower would have got payoff which were higher than the minimum payoff required to solve her incentive compatibility problem.

So, in this question we only find the lower bound on the borrower's rents. The borrower would get more than her lower bound depending on the her bargaining strength.

(d) *Would the lending efficiency of group lending increase if we reduced the relative bargaining strength of the lender?*

The lending efficiency is determined by the social surplus each project creates. This is obtained by taking the economic rents and opportunity cost of capital away from expected output of the project. Reducing the bargaining strength of the lender and increasing the bargaining strength of the lender would just influence the way in which the surplus is split between the lender and borrower. It does not change the surplus. So, the lending efficiency of group lending is not related to the relative bargaining strength of the lender.

3. In Ghatak and Guinnane (1999)'s enforcement model, the individual liability borrowing repayment condition is

$$u(x) - u(x - r) \leqslant B$$

where x is the output realisation of the project, r is the interest rate due and B is the net present discounted value of having continued access to credit in the future. The joint-liability group lending repayment condition is

$$u(x) - u(x - 2r) \leqslant B.$$

Assume that $B > 0$ and the utility function is logarithmic (to the base e), i.e., $u(x) = \log_e(x)$.

(a) *Show that under both types of lending arrangements, borrowers repay only if the output exceeds a certain threshold level. Which type of lending arrangement has a higher threshold?*

Individual Lending threshold:

$$\ln(x) - \ln(x - r) \leqslant B$$

$$\Rightarrow \quad x \geqslant r\left(\frac{e^B}{e^B - 1}\right) = x_{IL}$$

Joint Liability group lending threshold:

$$\ln(x) - \ln(x - 2r) \leqslant B$$

$$\Rightarrow \quad x \geqslant 2r\left(\frac{e^B}{e^B - 1}\right) = x_{JL}$$

Thus,

$$x_{JL} > x_{IL}$$

(b) *Find the output ranges over which the group lending does better in terms of repayment than individual lending and vice versa.*

Let x_i be the output realisations of individual i.

Joint liability does better if one group member is unwilling to repay and the other member fully meets the group's repayment obligations i.e., if $x_i < x_{IL}$ and $x_j > x_{JL}$.

Individual liability does better if one group member is unwilling/unable to repay and the other member is willing to repay her own debt but not meet the group's full repayment obligations[4] i.e., if $x_i < x_{IL}$ and $x_{IL} < x_j < x_{JL}$.

(c) *Explain how the repayment rate may improve if the group members are able to impose significant social sanctions on each other.*

Given that $x_{IL} = r\left(\frac{e^B}{e^B-1}\right)$, it follows that $x_{IL} > r$. That is when $x_i(r, x_{IL})$ the individuals default strategically.

Social sanction reduces the attractiveness of payoff stream when one members defaults strategically in the range $r < x_i < x_{IL}$ and the other member us willing to pay her own loan but not her partner's $x_{IL} < x_j < x_{JL}$. The threat of social sanction S lower's i's threshold to $x \geqslant r\frac{e^{(B+S)}}{e^{(B+S)}-1}$ and decreases the range over which she will default strategically.

(d) *If group maximises joint welfare in this model (as would be the case if and the repayment decisions are taken co-operatively), argue that repayment rates under joint liability will be identical to repayment rates under individual liability.*

If the group maximises joint welfare and repayment decisions are made cooperatively, the group acts like one individual taking two loans (and not two individuals taking two separate loans). Thus, for group's repayment decision, the individual (and not group) liability threshold applies. Put another way, the crucial variable in the determination of repayment decisions is not the individual income (as in the non-cooperative formulation) but the total average income.

References

Ghatak, M. and Guinnane, T. W. (1999). The economics of lending with joint liability: theory and practice. *Journal of Development Economics*, 60(1):195–228.

[4]Due to joint liability, she does not repay even her own debt because doing so does not save her from incurring a loss of B.