Adverse Selection

Credit & Microfinance

Dr. Kumar Aniket
University of Cambridge

Lecture 2

Borrower's Project & Type and so on

- Borrower's project

$$1 \text{ unit of capital } \longrightarrow \begin{cases} x_i & \text{with probability } p_i \\ 0 & \ldots\ (1-p_i) \end{cases}$$

- Borrower type $i = \{s, f\}$

$$\begin{cases} p_s & \text{(Safe type)} \\ p_r & \text{(Risky type)} \ \ldots\ p_r < p_s \end{cases}$$

- Borrower's type unobservable to lender

Environment

- Impoverished borrower i
  - Risk neutral
  - No wealth
  - Reservation utility is $\bar{u}$
  - proportion of risky type $r \rightarrow \theta$
    proportion of safe type $s \rightarrow (1-\theta)$
- Lender
  - Risk neutral
  - opportunity cost of capital ρ
  - Lends in a competitive loan market ... *lender's zero profit condition*

First Best: Perfect Information Benchmark

- If the lender knows borrower's type (perfect information environment) then the lender's profit condition would be:

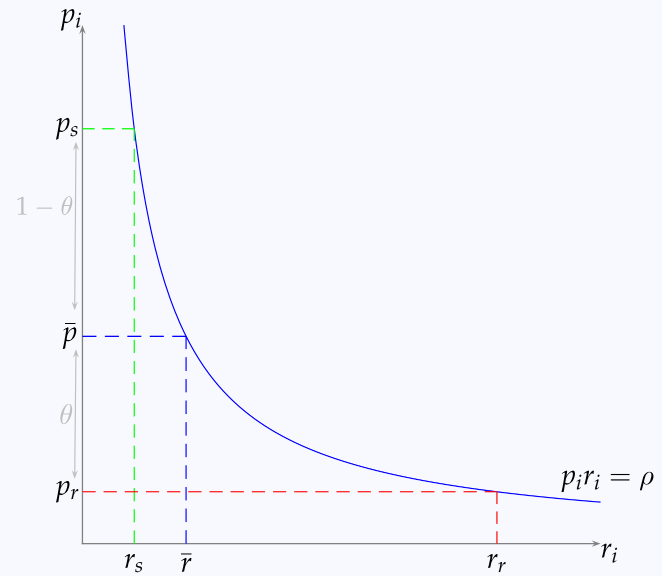
$$r_i = \frac{\rho}{p_i} \qquad i = r, s \qquad \text{(L-ZPC)}$$

... *lender charges r and s different rate*
... *risky type pays a higher interest rate*

- Borrower i's expected payoff

$$U_i(r) = payoff_i(x_i - r_i)$$

The borrower is risk neutral and thus only cares about her expected payoff.



Socially Viable Project

Socially Viable Project

A project is social viable if the expected output is greater than the social cost, in this case, the opportunity cost of capital and reservation wage in this case.

$$p_i x_i \geqslant \rho + \bar{u}$$

- Under perfect information, all socially viable projects are feasible.
  - The lender would offer the borrowers contracts contingent on their type and all borrowers'projects would be funded.

Second Best: Hidden Information Problem

If the lender is ignorant of the borrower's type, he has the following two options.

either lend to both type - Pooling Equilibrium

. . . both type pay the same pooling interest rate

$$\bar{p} = \theta p_r + (1-\theta) p_s \qquad \text{(loan repayment probability)}$$

$$\bar{r} = \frac{\rho}{\bar{p}} \qquad \text{(interest rate)}$$

or lend to only one type - Separating Equilibrium

. . . interest rate for the type left in the market

. . . Which type do you think this will be?

$$p_r \text{ or } p_s \qquad \text{(loan repayment probability)}$$

$$r_r = \frac{\rho}{p_r} \text{ and } r_s = \frac{\rho}{p_r s} \qquad \text{(resp. interest rates)}$$

Interest Rate

With the zero profit condition, we only have to check for three interest rates:

r_s – separating equilibrium with only the safe types

$\bar{r}$ – pooling equilibrium with both types

r_r – separating equilibrium with risky types . . .

Timeline:

Lender would choose the interest rate for the loan contract

Borrowers would choose whether to self-select in the loan contract

Imperfect Information: Adverse Selection

- Stiglitz & Wiess (1981)

$$p_s x_s = p_r x_r = \hat{x}$$

. . . the expected project outputs (mean) are identical

. . . the risky project has a greater spread around mean

- may lead to a problem of *Under-investment*
  some safe type with socially viable projects, i.e.,

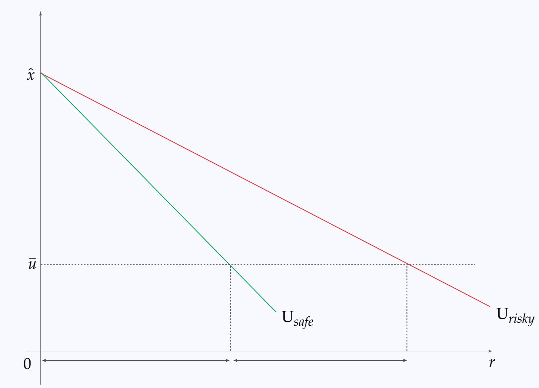
$$\hat{x} = p_s x_s \geqslant \bar{u} + \rho$$

. . . driven out of the loan market

Participation Constraint: Stiglitz & Wiess

Borrower's Participation Constraint

$$U_i(r_j) = \hat{x} - p_i r \geqslant \bar{u} \qquad i = r, s$$



Participation Constraint: Stiglitz & Wiess

Borrower's Participation Constraint

$$U_i(r_j) = \hat{x} - p_i r \geqslant \bar{u} \qquad i = r, s$$

- Check participation constraint for both types at $r_s, \bar{r}$ and r_s.
- Obtain lower threshold of $\hat{x}$ at which each type would self-select into the loan contract.

Interest rate	**Safe type**	**Risky type**
	$U_s(r) = \hat{x} - p_s r \geqslant \bar{u}$	$U_r(r) = \hat{x} - p_r r \geqslant \bar{u}$
$r_s = \frac{\rho}{p_s}$	$\hat{x} \geqslant \rho + \bar{u}$	$\hat{x} \geqslant \frac{p_r}{p_s}\rho + \bar{u}$
$\bar{r} = \frac{\rho}{\bar{p}}$	$\hat{x} \geqslant \frac{p_s}{\bar{p}}\rho + \bar{u}$	$\hat{x} \geqslant \frac{p_r}{\bar{p}}\rho + \bar{u}$
$r_r = \frac{\rho}{p_r}$	$\hat{x} \geqslant \frac{p_s}{p_r}\rho + \bar{u}$	$\hat{x} \geqslant \rho + \bar{u}$

Table: Self-selection condition at three interest rates in the Stiglitz Weiss

Under-Investment: Exclusion of the Safe type

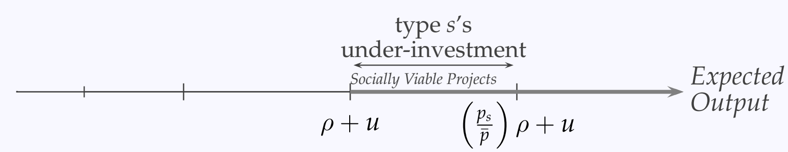


Figure: Safe type's under-investment project range

Under-investment: Some safe agents with socially viable projects i.e.,

$$\bar{u} + \rho < \hat{x} < \bar{u} + \frac{p_s}{\bar{p}}\rho$$

. . . unable to borrow.

Imperfect Information: Adverse Selection

- De Meza & Webb (1987)

$$p_s x > p_r x$$

... projects have different mean

... risky project has a lower mean

- may lead to a problem of *Over-investment*

  risky type with projects which are not social viable ($p_r x < \bar{u} + \rho$) may participate in the market at the pooling interest rate.

Participation Constraint: De Meza & Webb

Borrower's Participation Constraint

$$U_i(r) = p_i(x_i - r) \geqslant \bar{u} \qquad i = r, s$$

- Check participation constraint for both types at r_s, $\bar{r}$ and r_s.
- Obtain lower threshold of $\hat{x}$ at which each type would self-select into the loan contract.

Interest rate	Safe type $U_s(r) = p_s x - p_s r \geqslant \bar{u}$	risky type $U_r(r) = p_r x - p_r r \geqslant \bar{u}$
$r_s = \frac{\rho}{p_s}$	$p_s x \geqslant \rho + \bar{u}$	$p_r x \geqslant \frac{p_r}{p_s}\rho + \bar{u}$
$\bar{r} = \frac{\rho}{\bar{p}}$	$p_s x \geqslant \frac{p_s}{\bar{p}}\rho + \bar{u}$	$p_r x \geqslant \frac{p_r}{\bar{p}}\rho + \bar{u}$
$r_r = \frac{\rho}{p_r}$	$p_s x \geqslant \frac{p_s}{p_r}\rho + \bar{u}$	$p_r x \geqslant \rho + \bar{u}$

Table: Self-selection range at interest rates in the De Mezza Webb

Under-Investment: De Mezza & Webb

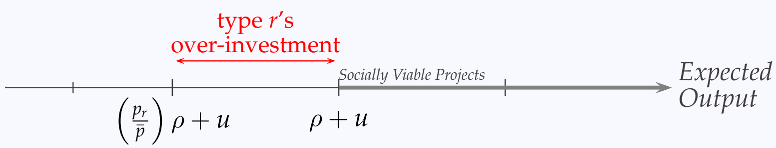


Figure: Risky type's over-investment project range

Over-investment: Risky type agents with projects that are not socially viable ($\bar{u} + \rho > p_r x > \bar{u} + \frac{p_r}{\bar{p}}\rho$) are able to borrow *(because they are cross-subsidised by the safe type borrowers).*

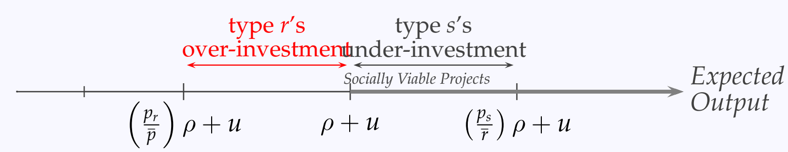


Figure: Under and Over investment Ranges

- **Under-investment:** *Range of socially viable projects that are not viable due to imperfect information*

$$\bar{u} + \rho < \hat{x} < \bar{u} + \frac{p_s}{\bar{p}}\rho$$

- **Over-investment:** *Range of socially non-Viable projects that are viable only due to imperfect information*

$$\bar{u} + \frac{p_r}{\bar{p}}\rho < p_r x < \bar{u} + \rho$$

Investment Problem in a Adverse Selection Framework

- Stiglitz & Webb
  Under-investment: Safe type unable to borrow for a range of socially viable projects because at high interest rates, only the risky types willing to borrow.
- De Meza & Webb
  Over-investment: Risky type are able to borrow for a range of non socially viable projects because they are cross-subsidised by the safe type borrowers in a pooling equilibrium.

Group Lending with Joint Liability

Definition (Joint-Liability Group-Lending)

Lender lends to a group with the proviso that each borrower's payoffs contingent on peer's outcome.

- Joint-Liability Group-Contract: (r, c)

Definition (Joint Liability Payment: c)

Payment due if the borrower succeeds but her peer fails

Definition (Positive Assortative Matching)

Groups homogenous in the types of borrowers

Positive Assortative Matching

Proposition (Positive Assortative Matching)

Joint Liability contracts lead to positive assortative matching.

$$\begin{aligned} U_{ij}(r,c) &= p_i p_j (x_i - r) + p_i(1-p_j)(x_i - r - c) \\ &= p_i(x_i - r) - p_i(1-p_j)c \end{aligned}$$

$$U_{rs}(r,c) - U_{rr}(r,c) = p_r(p_s - p_r)c \quad (1)$$

$$U_{ss}(r,c) - U_{sr}(r,c) = p_s(p_s - p_r)c \quad (2)$$

$$(2) > (1)$$

Positive Assortative Matching and Social Optimum

Paper (Ghatak, 1999, 2000)

Joint Liability Group Lending leads to positive assortative matching solves the problems of under and over-investment.

Assumption (Socially Optimal Matching)

Positive assortative matching maximises the aggregate expected payoffs of borrowers over all possible matches

$$U_{ss}(r,c) - U_{sr}(r,c) > U_{rs}(r,c) - U_{rr}(r,c) \quad ((2) > (1))$$

$$U_{ss}(r,c) + U_{rr}(r,c) > U_{rs}(r,c) + U_{rs}(r,c) \quad \text{(rearranging)}$$

Indifference Curves

Indifference Curve of borrower type i

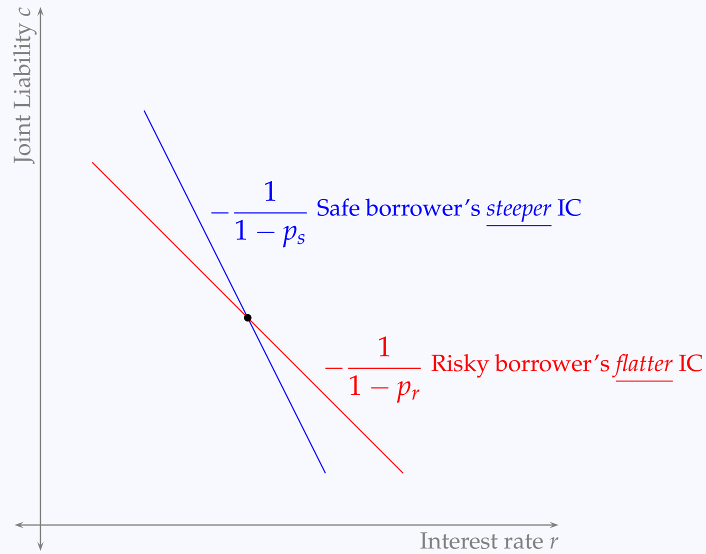
$$U_{ij}(r,c) = p_i(x_i - r) - p_i(1-p_j)c = \bar{k}$$

$$\left[\frac{dc}{dr}\right]_{U_{ii}=\text{constant}} = -\frac{1}{1-p_i}$$

s type's indifference curve steeper

$$\left|-\frac{1}{1-p_s}\right| > \left|-\frac{1}{1-p_r}\right|$$

Indifference Curves of the two types



Lender's Problem

- Lender offers group contracts (r_r, c_r) and (r_s, c_s) which maximise the borrower's payoff subject to the following constraint"s:

$$r_r p_r + c_r(1-p_r)p_r \geqslant \rho \quad \Rightarrow \quad \frac{dc}{dr} = -\frac{1}{1-p_r} \qquad \text{(L-ZPC}_r\text{)}$$

$$r_s p_s + c_s(1-p_s)p_s \geqslant \rho \quad \Rightarrow \quad \frac{dc}{dr} = -\frac{1}{1-p_s} \qquad \text{(L-ZPC}_s\text{)}$$

$$U_{ii}(r_i, c_i) \geqslant \bar{u}, \qquad i = r, s \qquad \text{(PC}_i\text{)}$$

$$x_i \geqslant r_i + c_i \qquad i = r, s \qquad \text{(LLC}_i\text{)}$$

$$U_{rr}(r_r, c_r) \geqslant U_{rr}(r_s, c_s) \qquad \text{(ICC}_{rr}\text{)}$$

$$U_{ss}(r_s, c_s) \geqslant U_{ss}(r_r, c_r) \qquad \text{(ICC}_{ss}\text{)}$$

Abbreviations

L-ZPC$_i$ Lender's Zero Profit Condition for type i

PC$_i$ Participation Constraint for type i

LLC$_i$ Limited Liability Constraint for type i

ICC$_{ii}$ Incentive Compatibility Constraint for group i, i

Separating Equilibrium in Group Lending

⊙ (L-ZPC$_s$) and (L-ZPC$_r$) cross at $(\hat{r}, \hat{c})$

Proposition (Separating Equilibrium)

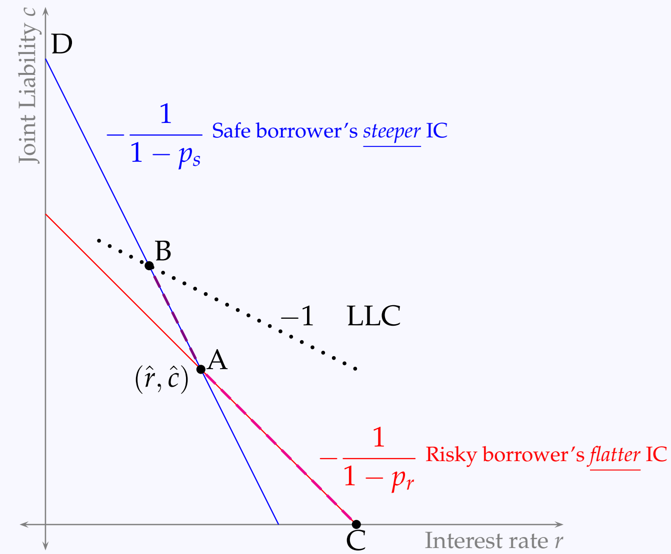
For any joint liability contract (r, c)

i. *if* $r_s < \hat{r}, c_s > \hat{c}$, *then* $U_{ss}(r_s, c_s) > U_{rr}(r_s, c_s)$

ii. *if* $r_r > \hat{r}, c_r < \hat{c}$, *then* $U_{rr}(r_r, c_r) > U_{ss}(r_r, c_r)$

- Safe groups prefer *high* joint liability payment *low* interest rates
- Risky groups prefer low joint liability payments high interest rate
- Different interest rates for different types – back to the perfect information environment

Separating Equilibrium in r-c Space



Contracts

Separating Contract

- Safe: Segment BA
- Risky: Segment AC

Pooling Contract

- $(\hat{c}, \hat{r})$ at A

Conditions: Projects sufficiently productive to satisfy the Limited Liability Condition (LLC) along respective contract segments.

Under-investment:

Bring back the safe borrowers with socially productive investment.

Over-investment:

Risky borrowers with socially productive investment drop out.

Conclusions

In group lending

- joint liability leads to positive assortative matching
  - the risky and safe group differ in the way they trade-off interest rates and joint liability payments
  - lender is able to discriminate between the risky and safe groups
- problem of under and over investment is solved