ADVERSE SELECTION
PAPER 8: CREDIT AND MICROFINANCE

LECTURE 2

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Abstract. We explore adverse selection models in the microfinance literature. The traditional credit market failure of under and over investment in individual lending loan contracts are explained. In group lending, a joint liability contract induces positive assortative matching within the group. Further, joint liability contracts can achieve the first best by solving the problems of under and over investment.

Major changes and revisions in Section 3.3 and 3.4.

1. Introduction

In this lecture, we look at the problem of private information. The potential borrowers are socially connected and live in a informationally permissive environment, where they know themselves and each other very well. The lender is not part of this information network and thus does not have access to the borrowers' information network.

The lender can use contracts to extract this information. The lecture explores one specific type of contract which would bind people together in groups, allowing the lender to extract the information from the social network. The resultant contract would be an improvement over the traditional individual lending contracts.

1.1. **Classification of Information Problem.** The potential borrowers differ in their respective inherent characteristics or ability to execute projects. We interpret these characteristics as the ones that determine the borrower's chances of successfully completing the project. The generic name for these characteristic is the *type*. In a credit market transaction, any information or characteristic about the borrower that affects the outcome is considered the type of the borrower. This is typically information that the lender would like to acquire before he lends to the borrower. The problem of acquiring this information is referred to as the adverse selection problem.

Once borrower obtain the loans, her *actions* like effort decision or project selection affect the outcome of the project. The lender would like influence these action to ensure that the borrower takes actions that are in the interest of the lender. The problem of influencing the borrower's action is called moral hazard. In this lecture, we confine ourselves to the adverse selection problem and would take up the moral hazard problem in the next lecture.

2. Model

We assume that borrowers are fully aware of their own characteristics as well as the characteristics other borrowers around them. That is the information environment is such that borrowers have perfect information about each other. Of course, there is an information partition between the lender and borrowers such that lender does not know the borrower's type.

The lender's problem thus is that the borrowers posses some private or hidden information, which is relevant to the the project. The lender would like to extract this information. The only way he can do that is through the loan contracts he offers the borrowers. We set out the main ideas in the context of the wider adverse selection literature. We then examine how the lender can extract information inexpensively through group lending, i.e., by offering inter-linked contracts to multiple borrowers simultaneously.

The main mechanism we explore in this lecture is joint liability. The lender could offer the contract to a group of borrowers in stead of offering it to individuals. This would allow him to inter-link the borrower's payoffs by making it contingent on her own as wells as the payoff of her peer. The part of the payoff that is contingent on her peer's outcome is the joint liability component of the payoff. We show that this joint liability component is critical in dissuading the wrong kind of borrower and encouraging the right kind of borrower to borrow from the lender.

2.1. **The Principal-Agent Framework.** We use the principal agent framework to analyse the problem of lending to the poor. Usually, a principal is the uninformed party and the agent the informed party, the party possessing the private or hidden information. This information needs to have a bearing on the task the principal wants to delegate to the agent. The information gap between the principal and the agent has some fundamental implication for the bilateral or multi-lateral contract they may choose to sign. Further, even though the agent(s) may renege on their contract, the assumption always is that the principal never does so.

In the context of the credit markets, the term principal is used interchangeably with lender and the term agent is used interchangeably with borrower. Unless stated otherwise, we assume throughout the rest of the lectures that the lender and the borrower(s) are both risk-neutral.

2.2. **Project.** A project requires an investment of 1 unit of capital and at the start of period 1 and produces stochastic output x at end of period 1. All borrowers have zero wealth and can thus only initiate the project if the lender agrees to lend to her.

Explanation: This is a way of introducing the limited liability clause, which ensures that the borrower's liability from a loan contract is limited to the output of the project. The lender does not acquire wealth from the borrower ex post if the project fails. To make the distinction clear, collateral is the wealth acquired by the lender before the lending starts. Some lenders, especially the informal ones, may have the ability to force the borrower to give up wealth after the borrower has defaulted on the loan. As we discussed in the last lecture, the limited liability clause maybe realistic when describing the borrower's interaction with an formal lender, who is from outside the social network, but may not be realistic when describing the borrower's interaction with the local informal lenders.

As is typical in a adverse selection model, the value, as well as the stochastic property[1] of the output depends on the type of borrower undertaking the project. To keep matters simple, we assume that the project produces a output with strictly positive value when it succeeds and zero when it fails.

A project undertaken by a borrower of type i produces an output valued at x_i when it succeeds and 0 when it fails. Further, the probability of the project succeeding is contingent on the borrower types. The project succeeds and fails with probability p_i and $1 - p_i$.

The Agents. We have a world with two types of agents or borrowers, the *safe* and the *risky* type. The projects that risky and safe types' undertake succeed with probability p_r and p_s respectively with $p_r < p_s$. That is, the risky type succeeds less often then the safe type. The proportion of risky type and safe type is θ and $1 - \theta$ respectively in the population. The expected payoff of an agent of type i is given by

$$U_i(r) = p_i(x - r).$$

Given that interest is paid only when the agents succeed, the safe agent's utility is more interest sensitive as compared to the risky agent's utility since she succeeds more often.[2] Both types are impoverished with no wealth and have a reservation wage of $\bar{u}$.

[1]The probabilities associated with the various outcomes

[2]As we see in the section on group lending, this leads to the safe types utility having a steeper slope than the risky types in the figures ahead.

The Principal. The principal's or the lender's opportunity cost of capital is ρ, i.e., he either is able to borrow funds at interest rate ρ to lend on to his clients or has an opportunity to invest his own funds in a risk-less asset which yields a return of ρ.

We assume that the lender is operating in a competitive loan market and can thus can make no more than zero profit. This implies that the lender lends to the borrowers at a *risk adjusted interest rate*. The lender's zero profit condition $\rho = p_i r$ ensures that on a loan that has a repayment rate of p_i, the interest rate charged is always

$$r_i = \frac{\rho}{p_i} \tag{1}$$

It is important to note that competition amongst the lenders ensures that a particular lender can only choose whether or not to enter the market. He is not able to explicitly choose the interest rate he lends at. He always has to lend at the risk adjusted interest rate, at which he makes zero profits. Given that p_r, p_s, θ and ρ are exogenous variables, we can take the respective risk adjusted interest rate to be exogenously given as well.

In the lecture on moral hazard we discuss the conditions under which making the assumption of zero profit condition would be justified. We find that this assumption is not critical at all. What matters is the surplus that a project creates. The assumptions on conditions in the loan market just determine the way in which this surplus is shared between the lender and the borrower.

2.3. **Concepts.**

2.3.1. *Repayment Rate.* The repayment rate on a particular loan is the proportion of borrowers that repay back.[3] If the lender is able to *ensure* that he lends only to the risky type, his repayment rate is p_r. Similarly, it is p_s if he only lends to the safe type. If he lends to both type, his average repayment rate is $\bar{p} = \theta p_r + (1-\theta)p_s$.

2.3.2. *Pooling and Separating Equilibrium.* If the lender is not able to distinguish between two types of agents, then the only way in which he can discriminate between the two types is by inducing them to self select and reveal their hidden information.

In a pooling equilibrium, both types of agents accept the same loan contract. Consequently, both types of agents are pooled together under the same loan contract. Conversely, in a separating equilibrium, a particular loan contract is accepted by only one type. The lender is able to induce the agents to reveal their private information by self selecting into different types of loan contracts.

2.3.3. *Socially Viable Projects.* Socially viable projects are the ones where the output exceeds the opportunity cost of labour and capital involved in the project.

$$p_i x \geqslant \rho + u \qquad i = r, s; \tag{2}$$

That is the expected output of the project exceeds the reservation wage of the agent and the opportunity cost of capital invested in the projects. In an ideal (read first best) world, all the socially viable projects would be undertaken and that lays the perfect information bench mark for us. What is of interest to us is how the problems associated with imperfect information restrict the range of projects that remain feasible. That is the extent to which some socially viable projects would not be feasible with imperfect information.

3. Individual Lending

In this section we look at individual lending and explore the implication of hidden information on the optimal debt contracts offered by the lender to the borrower.

[3]Put another way, given the past experience, it is also the lender's bayesian undated probability that the borrowers of future loans would repay.

3.1. **First-Best.** In the first best or the *perfect information* world, the lender can identify the type he is lending to and can tailor the contract accordingly. Consequently, he would lend to the safe type at the interest rate $r_s = \frac{\rho}{p_s}$ and to the risky type at the interest rate $r_r = \frac{\rho}{p_r}$. Given that $p_r < p_s$, i.e., the risky type succeeds and repays back less often, the risky type gets the loan at a higher interest rate as compared to the safe type. (Figure 1)

The utility of an agent type i that obtains the loan at her type specific interest rate r_i is given by the following expression.

$$U_i(r_i) = p_i x_i - p_i r_i \geqslant \bar{u}$$

Using (L-ZPC), this can be rearranged to show that all social projects get financed with perfect information.

$$p_i x_i \geqslant \rho + \bar{u}$$

3.2. **Second-Best.** In absence of the ability to discriminate between the risky type and the safe type agents, the lender has no option but to offer a single contract. This contract may either attract both types (pooling equilibrium) or just attract one of the two types (separating equilibrium).

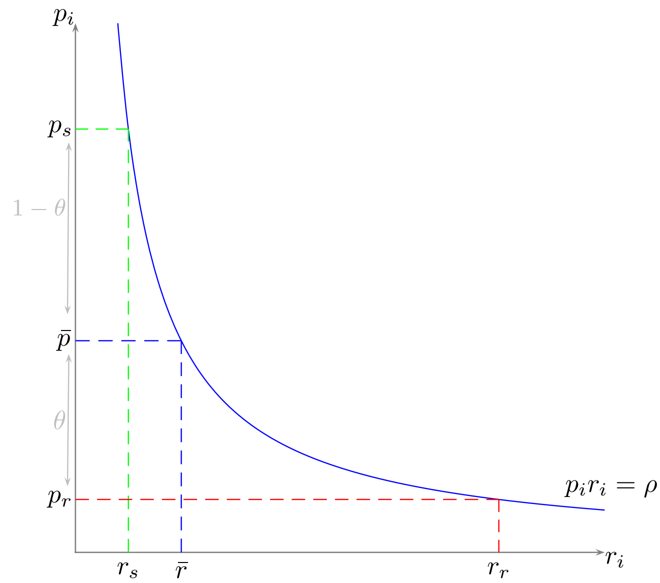

FIGURE 1. Perfect Information Benchmark

3.2.1. *Implication of Zero Profit Condition.* The loan market is competitive and consequently profits are driven down to zero. Interest Rates on the loan contract is always set according to the Zero Profit Condition (L-ZPC) in the loan market.

$$r_i = \frac{\rho}{p_i} \tag{L-ZPC}$$

There are only the following three interest rate possible in this scenario.

$$r_s = \frac{\rho}{p_s}$$
$$r_r = \frac{\rho}{p_r}$$
$$\bar{r} = \frac{\rho}{\bar{p}}$$

r_s is the interest rate charged if the lender is convinced that the only *safe* agents would take up the contract. Similarly, the lender would charge r_r and if he is convinced that only the *risky* borrowers would self select into the loan contract. If both types would self select into the contract, the lender would charge $\bar{r} = \frac{\rho}{\bar{p}}$ where $\bar{p} = \theta p_r + (1-\theta)p_s$. Since $p_r < \bar{p} < p_s$ it follows that

$$r_s < \bar{r} < r_r \tag{3}$$

Under the assumption of zero profit condition, the lender would not charge any other interest rate. Thus, we examine how each type self selects in or out of the loan contracts at the each of these three interest rates.

3.2.2. *Contract Space.* The lender can either offer a contract that is targeted towards a specific type or could offer a contract that induces both type in the borrowing pool. For risky and safe type, the interest rate is the risk adjusted interest rate $r_r = \frac{\rho}{p_r}$ and $r_s = \frac{\rho}{p_s}$ respectively. If the borrowing pool has both types, the lender's average or pooling repayment rate across his cohort of risky and safe borrowers is given by

$$\bar{p} = \theta p_r + (1-\theta)p_s \tag{4}$$

In this case, the interest rate would be $\bar{r} = \frac{\rho}{\bar{p}}$. The lender's contract space is $[r_s, r_r]$ given that $r_s \leqslant \bar{r} \leqslant r_r$.

3.2.3. *The Constraints.* The lender has to makes sure that any contract that he offers satisfies the following conditions.

(1) *Participation Constraint:* This condition is satisfied if the lender provides the borrower sufficient incentive to accept the loan contract, i.e., participate in the loan transaction.[4]

$$U_i(r_r, \dots) \geqslant \bar{u}$$

(2) *Incentive Compatibility Constraint:* In a separating equilibrium, the incentive compatibility condition is satisfied if each borrower type has the incentive to take the contract meant for her and does not have any incentive to pretend to be the other type. These conditions are as follows.

$$U_r(r_r, \dots) > U_r(r_s, \dots)$$
$$U_s(r_s, \dots) > U_s(r_r, \dots)$$

The $\dots$ just denote the additional variables that the lender could specify in the contract, which would help in getting these constraints satisfied.

Explanation: Lets explore this further and say that the lender's contract has two components, the interest rate r and some other component ϑ. The lender can now offer two contracts. He can offer a contract (r_r, ϑ_r) meant for the risky type and a contract (r_s, ϑ_s) for the safe type. We would get a separtating equilibrium if the following conditions hold.

$$U_r(r_r, \vartheta_r) > U_r(r_s, \vartheta_s)$$
$$U_s(r_s, \vartheta_s) > U_s(r_r, \vartheta_r)$$

The first equation just says that the risky type strictly prefers taking the contract meant for her, that is she prefers taking that contract (r_r, ϑ_r) over a alternative contract (r_s, ϑ_s). Similarly, the second equation is satisfied when the safe type strictly prefers taking the contract (r_s, ϑ_s) over one the alternative one (r_r, ϑ_s).

Of course this would only work if ϑ_i entered the borrower's utility function. If it did not, the lender would be left with a contract that effectively only specifies the interest rate r and thus the lender would be offering only one interest rate to both types.[5] At this interest rate, either both types would accept the contract leading to a pooling equilibrium or only one type would accept the contract leading to a separating equilibrium.

[4]This is also called the Individual Rationality Condition.

[5]If the lender offered two interest rates, all rational borrowers would choose the lower one.

(3) *Break even condition:* Break-even condition is the lower bound on the profitability, that is, the lender's profit should not be less than zero. Turns out the competition in the loan market puts an upper bound on profits and ensures that profits cannot be more than zero. This is called the *zero profit condition.* Thus, in this case the lender's break even condition and zero profit condition give us a condition that binds with equality.

Turns out, the precise course of action the lender would take depends on the stochastic properties of project. Specifically, it depends on the first and second moments of the expected output of the project.

3.3. **The Under-investment Problem.** Stiglitz and Weiss (1981) analyse the problem under the assumption that both types' project have the same expected output and the risky type produces an output of a higher value than the safe type since he succeeds less often.

$$p_r x_r = p_s x_s = \hat{x} \tag{5}$$
$$p_r < p_s \;\Rightarrow\; x_r > x_s$$

The assumption in the Stiglitz Weiss model is that each type has the *same expected output*[6] $\hat{x}$ from the project, but since safe type succeeds more often than the risky type, the safe types output is accordingly smaller than the risky type's output. The borrower i's utility is as follows:

$$U_i(r) = \hat{x} - p_i r$$

where $\hat{x} = p_s x_s = p_r x_r$. A borrower of type i with the outside option of $\bar{u}$ would only take up the loan contract if the following condition is met.

$$U_i(r) = \hat{x} - p_i r \geqslant \bar{u} \tag{6}$$

Interest rate	**Safe type**	**Risky type**
	$U_s(r) = \hat{x} - p_s r \geqslant \bar{u}$	$U_r(r) = \hat{x} - p_r r \geqslant \bar{u}$
$r_s = \frac{\rho}{p_s}$	$\hat{x} \geqslant \rho + \bar{u}$	$\hat{x} \geqslant \frac{p_r}{p_s}\rho + \bar{u}$
$\bar{r} = \frac{\rho}{\bar{p}}$	$\hat{x} \geqslant \frac{p_s}{\bar{p}}\rho + \bar{u}$	$\hat{x} \geqslant \frac{p_r}{\bar{p}}\rho + \bar{u}$
$r_r = \frac{\rho}{p_r}$	$\hat{x} \geqslant \frac{p_s}{p_r}\rho + \bar{u}$	$\hat{x} \geqslant \rho + \bar{u}$

TABLE 1. Self-selection condition at the three interest rates in the Stiglitz Weiss Model

Table 1 gives us the condition under which the borrower of a particular type would self select into the a loan contract at the each of the three possible interest rates. Below we look at the potential pooling and separating equilibria at each of the three interest rates. Remember that the lender would satisfy the zero profit condition only under the following three conditions:

(1) Separating equilibrium with on the *safe* type at r_s
(2) Pooling equilibrium with both the safe and risky type at $\bar{r}$
(3) Separating equilibrium with on the *risky* type at r_r

In the discussion below, we will look for condition under which these three equilibria can be achieved. Remember that both type have the same expect project output $\hat{x}$ and they just differ in the probability of success.

(1) *The lender could offer loan contracts at r_s if there exists a separating equilibrium with only safe types. Any other equilibrium would lead to negative profits for the lender.*

[6]This is also called the Second Order Stochastic Dominance, i.e., one project dominates the other one in the second moment and not the first moment.

If the lender offers loans at r_s, the safe type would self-select into the contract if the expected output is $\hat{x} \geqslant \rho + \bar{u}$ and the risky type would self-select into the contract if the expected output is $\hat{x} \geqslant \frac{p_r}{p_s}\rho + \bar{u}$.[7]

If the expected output of the project is in the range $\hat{x} \geqslant \rho + \bar{u}$, then both the safe and risky type would self-select into the contract leading to a pooling equilibrium. If the expected output of the project is in the range $\rho + \bar{u} > \hat{x} \geqslant \frac{p_r}{p_s}\rho + \bar{u}$, then only the risky type would self-select into the contract leading to a separating equilibrium for the risky type.

In the first case, with $\hat{x} \geqslant \rho + \bar{u}$, a pooling equilibria at r_s would lead to the lender making negative profits. Similarly, with $\rho + \bar{u} > \hat{x} \geqslant \frac{p_r}{p_s}\rho + \bar{u}$ in the second case, a separating equilibrium with the risky type would lead to negative profits for the lender. *Consequently, there are no equilibria at r_s that would allow the lender to make non-negative profits.*

(2) *The lender could offer a loan contract at $\bar{r}$ if either there exist a pooling equilibrium with both types or a separating equilibrium with the safe types. Any other contract, i.e., a separating equilibrium with only risky types would lead to negative profits for the lender.*

At $\bar{r}$, the safe type self-selects into the loan contract if $\hat{x} \geqslant \frac{p_s}{\bar{p}}\rho + \bar{u}$ and the risky type self-selects into the loan contract if $\hat{x} \geqslant \frac{p_r}{\bar{p}}\rho + \bar{u}$. Again, there are two relevant ranges. In the range $\hat{x} \geqslant \frac{p_s}{\bar{p}}\rho + \bar{u}$, both types would self-select leading to a pooling equilibrium. In the range $\frac{p_s}{\bar{p}}\rho + \bar{u} > \hat{x} \geqslant \frac{p_r}{\bar{p}}\rho + \bar{u}$, only the risky type would self select in.

The lender would achieve a **pooling equilibrium** with non-negative profits in the range $\hat{x} \geqslant \frac{p_s}{\bar{p}}\rho + \bar{u}$. Conversely, a separating equilibrium in with only the risky type in the range $\frac{p_s}{\bar{p}}\rho + \bar{u} > \hat{x} \geqslant \frac{p_r}{\bar{p}}\rho + \bar{u}$ would lead to negative profits. *Consequently, the only feasible option is a pooling equilibrium if the output is in the range $\hat{x} \geqslant \frac{p_s}{\bar{p}}\rho + \bar{u}$.*

(3) *At r_r, any of the three contracts allow the lender to make non-negative profits. The lender could offer a loan contract at r_r if there exists a pooling equilibrium with both types or a separating equilibrium with either the safe or the risky type.*[8]

At r_r, the safe type would self-select into the loan contract if $\hat{x} \geqslant \frac{p_s}{p_r}\rho + \bar{u}$ and the risk type would self-select into the contract if $\hat{x} \geqslant \rho + \bar{u}$. In the expected output range $\hat{x} \geqslant \frac{p_s}{p_r}\rho + \bar{u}$, the lender would achieve a **pooling equilibrium** and in the expected output range $\frac{p_s}{p_r}\rho + \bar{u} > \hat{x} \geqslant \rho + \bar{u}$ the lender would achieve a **separating equilibrium** with only the **risky type**. *Both the pooling equilibrium in the range $\hat{x} \geqslant \frac{p_s}{p_r}\rho + \bar{u}$ and separating equilibrium with risky type in the range $\frac{p_s}{p_r}\rho + \bar{u} > \hat{x} \geqslant \rho + \bar{u}$ would allow the lender to make non-negative profits.*

To summarise, there are three ranges $\hat{x}$ in which a equilibrium could exist.

- In the range $\hat{x} \geqslant \frac{p_s}{\bar{p}}\rho + \bar{u}$ there exist a pooling equilibrium at the interest rate $\bar{r}$.
- In the range $\frac{p_s}{\bar{p}}\rho + \bar{u} > \hat{x} \geqslant \frac{p_s}{p_r}\rho + \bar{u}$, there exist a pooling equilibrium at interest rate r_r.
- In the range $\frac{p_s}{p_r}\rho + \bar{u} > \hat{x} \geqslant \rho + \bar{u}$ there exist a separating equilibrium with only the risky type at interest rate r_r.

Lets look at it from the perspective of the borrower type.

- The risky type obtains the loan if the expected output is in the range $\hat{x} \geqslant \rho + \bar{u}$. This implies that for the risky type, all the socially viable projects get financed.
- The safe type only obtains the loan if the expected output is in the range $\hat{x} \geqslant \frac{p_s}{p_r}\rho + \bar{u}$. For the safe type, some socially viable projects in the range of $\frac{p_s}{p_r}\rho + \bar{u} > \hat{x} \geqslant \rho + \bar{u}$ does not financed, leading to under-investment in the credit market.

The interesting thing to notice is that as interest rate goes up, the threshold value of the expected output $\hat{x}$ beyond which the two type would self-select into the contract always rises faster for the safe type than for the risky type. As a result, as the interest rate increases, the pool of applicant worsens

[7] Since the $p_r < p_s$, the expected output threshold for the risky borrower is lower than the threshold for the safe borrower. We find that the thresholds for the risky and the safe borrower increase with the interest rate, though risky borrower's threshold always smaller than the safe borrower's threshold.

[8] In the pooling equilibrium or the separating equilibrium with the safe type, the lender would make positive profits. In the separating equilibrium with the risky type, the lender would zero profit.

for the lender. This is because of the principle of limited liability for the borrower. From (6) we can see that the borrowers only pay back when the project succeeds. Since the risky type succeeds less often, she self-selects into the loan contract at lower $\hat{x}$.

In any normal market, the price is the equilibrating mechanism. In this case, as the price (interest rate) rises, the pool of loan applicant worsens and as a result price (interest rate) no longer works as an equilibrating mechanism. Consequently, some safe borrowers with socially viable projects are not able to borrow in this market.

Under-Investment: Ideally all socially viable projects ($\hat{x} \geqslant \rho + \bar{u}$) should be financed. From the above analysis, we can see that a safe type would never be able to get a loan if her project is in the range $\frac{p_s}{\bar{p}}\rho + \bar{u} > \hat{x} \geqslant \rho + \bar{u}$. We have a lower bound on the safe type's expected output of the project that gets financed.

$$\hat{x} \geqslant \frac{p_s}{\bar{p}}\rho + u. \tag{7}$$

Since $p_s > \bar{p}$,[9] we find that there are safe type's projects that would not be financed even though they are socially viable.[10]

$$\hat{x} \in \left[\left(\frac{p_s}{\bar{p}}\right)\rho + u, \quad \rho + u,\right]$$

In this range, the lender would lend only lend to the risky type in a separating equilibrium. Consequently, the *under-investment problem* in Stiglitz and Weiss (1981) is that there are some safe type's project that do not get financed even though they are socially viable. In terms of their productivity, these projects are on the lower end of the socially viable projects. They are below the threshold level defined by (7) but above the threshold given by (2). Conversely, all risky type's socially viable projects get financed.

3.4. **The Over-investment Problem.** De Mezza and Webb (1987) analyse the case when the two types produce identical outputs when they succeed. Consequently, the safe type's project has a higher productivity than the risky type's project.[11]

$$p_r\bar{x}_< p_s\bar{x} \tag{8}$$

It follows that for an interest rate in the relevant range, the safe type's payoff is always higher than the risky type's payoff.

$$U_s(r) > U_r(r) \qquad \forall\, r \in [0, \bar{x}];$$

The assumption in the De Mezza model is that each type has the same output of the project, but the safe type succeeds more often than the risky type ($p_s > p_r$) and thus has a higher expected output. $p_s x > p_r x$. The borrower i's utility is as follows:

$$U_i(r) = p_i x - p_i r$$

where the output for both types is constant at x when they succeed and 0 when they fail. A borrower of type i with the outside option of $\bar{u}$ would only take up the loan contract if the following condition is met.

$$U_i(r) = p_i x - p_i r \geqslant \bar{u}$$

Table 2 gives us the condition under which the borrower of a particular type would self select into the a loan contract at the each of the three possible interest rates. As interest rate increases from r_s to r_r,

[9]The pooling repayment rate is a weighted sum of risky and safe type's respective repayment rates and thus would always be lower than the higher of the two repayment rates, the safe type's repayment rate.

[10]Note that the projects that are not financed are on the lower end of the productivity scale. If the projects are productive enough, all socially viable projects get financed.

[11]This is the case under which you get first order stochastic dominance. That is, one project dominates the other project in first moment.

the minimum expected output required to self select in the loan contract increases. The safe type start from the socially viable projects and the risky types start from projects that are non socially viable.

Interest rate	Safe type $U_s(r) = p_s x - p_s r \geqslant \bar{u}$	risky type $U_r(r) = p_r x - p_r r \geqslant \bar{u}$
$r_s = \frac{\rho}{p_s}$	$x \geqslant \frac{\rho}{p_s} + \frac{\bar{u}}{p_s}$	$x \geqslant \frac{\rho}{p_r} + \frac{\bar{u}}{p_r}$
$\bar{r} = \frac{\rho}{\bar{p}}$	$x \geqslant \frac{\rho}{\bar{p}} + \frac{\bar{u}}{p_s}$	$x \geqslant \frac{\rho}{\bar{p}} + \frac{\bar{u}}{p_r}$
$r_r = \frac{\rho}{p_r}$	$x \geqslant \frac{\rho}{p_s} + \frac{\bar{u}}{p_s}$	$x \geqslant \frac{\rho}{p_r} + \frac{\bar{u}}{p_r}$

Table 2. Self-selection range at the three interest rates in the De Mezza Webb Model

- **Over-investment:** If $\frac{p_r}{\bar{p}}\rho + \bar{u} < p_r x < \rho + \bar{u}$, the lender would get a pooling equilibrium with *over-investment* as risky types invest with project that are not socially viable, i.e., $p_r x < \rho + \bar{u}$.[12]
- For a pooling equilibrium to exist at $\bar{r}$, both borrowers should self select into the contract. This would happen if the following condition on x is satisfied. $x \geqslant \max\left[\left(\frac{\rho}{\bar{p}} + \frac{\bar{u}}{p_s}\right), \left(\frac{\rho}{\bar{p}} + \frac{\bar{u}}{p_r}\right)\right]$.

  Since $p_s > p_r$, we find that a pooling equilibrium would exist if $x \geqslant \left(\frac{\rho}{\bar{p}} + \frac{\bar{u}}{p_r}\right)$. In this pooling equilibrium, both the risky and the safe type would self-select into the contract. Note that the risky type's projects are not socially viable since $p_r x \geqslant \left(\frac{p_r}{\bar{p}}\rho + \bar{u}\right)$ and $\rho + \bar{u} > \left(\frac{p_r}{\bar{p}}\rho + \bar{u}\right)$.

  Further, for the safe type, $p_s x \geqslant \left(\frac{p_s}{\bar{p}}\rho + \bar{u}\right) \geqslant \rho + \bar{u}$, which means some socially viable projects for the safe type are not financed in this pooling equilibrium.
- In the pooling equilibrium, if it exists, the risky types are cross-subsidised by the safe types.

3.4.1. *Existence of Pooling equilbrium.* We have shown above that $r_s < \bar{r} < r_r$. From the diagram in the lectures we can see that for separating equilibrium for safe type to exist we would need a very high r_s. Higher the r_s, higher the $\bar{r}$. This in turn would mean if the separating equilibrium exists a pooling equilibrium cannot exist, since $\bar{r}$ would always be higher than r_s. Lets look for the condition for a pooling equilibrium to exist keeping in mind that if pooling equilibrium exists, a separating equilibrium for the safe type cannot exist.

The risky type would take the loan contract if

$$p_r x \geqslant p_r r + \bar{u}$$

From this it follows that the risky type would take on a loan contract if the interest rate is in the following range.

$$r \leqslant x - \frac{\bar{u}}{p_r}$$

If $\bar{r}$ falls in this range, the pooling equilibrium would exist and since $r_s < \bar{r}$, no separating equilibrium can exist. The pooling equilibrium would exist if

$$\bar{r} \leqslant r \leqslant x - \frac{\bar{u}}{p_r}$$

Using $\bar{r} = \frac{\rho}{\bar{p}}$, we write the above condition as

$$p_r x \geqslant \frac{p_r}{\bar{p}}\rho + \bar{u} \tag{9}$$

This expression turns out to be the lower bound for our over-investment range. If this condition does not hold, then we can get a separating equilibrium.

[12] $\frac{p_s}{\bar{p}}\rho + \bar{u} < p_r x$ comes from the condition that ensures the existence of the pooling equilbrium and ensures that a separating equilibrium for the safe types cannot exists. See below.

3.4.2. *Discussion.* The result we get here is because we are imposing *perfect competition* in the loan market through the *zero profit condition* (L-ZPC). This does give the lender any flexibility in setting the interest rate. We thus have to consider the self selection and the zero profit condition together.

The over-investment problem in De Mezza and Webb (1987) is that there are risky type's projects that are financed even though they are not socially viable and have a negative impact on the social surplus. This happen because the lender is not able to discriminate between a borrower of a safe and risky type due to the hidden information they posses. The over-investment projects are the ones that do not satisfy the socially viability condition defined by (2) and are yet above the threshold defined by (9) which allows them to satisfy the risky type's participation constraint. The under and over-investment problem is summarised in Figure 2.

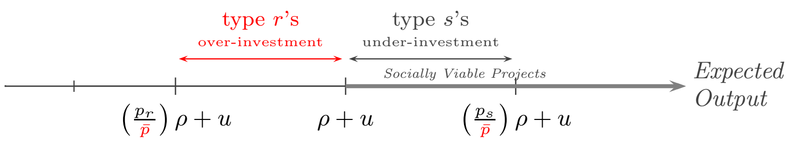

FIGURE 2. Under and Over investment Ranges

4. Group Lending with Joint Liability

This section is a simplified version of Ghatak (1999) and Ghatak (2000). The lender lends to borrowers in groups of two. The contract that the lender offers the group is such that the final payoffs are contingent on each other's outcome. Consequently, the members within the group are jointly liable for each other's outcome. If a borrower succeeds, she pays the specified interest rate r. Further, if her peer fails, she is required to pay an pay an additional joint liability component c. The lender offers a joint liability contract (r, c) where he specifies

- r: The interest rate on the loan due if the borrower succeeds.
- c: The additional joint liability payment which is incurred if the borrower succeeds but her peer fails.

Of course, if a borrower's project fails, the limited liability constraint applies and the borrower does not have a pay anything. A borrower's payoff in the group lending is given by.

$$\begin{aligned} U_{ij}(r,c) &= p_i p_j (x_i - r) + p_i(1-p_j)(x_i - r - c) \\ &= p_i(x_i - r) - p_i(1-p_j)c \end{aligned}$$

With probability p_i, the borrower succeeds. If she succeeds, she repays r and keeps $(x_i - r)$ for herself. With proability $p_i(1-p_i)$, she succeeds but her peer fails. In this case she has to make the joint liability payment c. Given the group contract (r, c) on offer, lender requires that the borrowers self-select into groups of two before they approach him for a loan.

4.1. **Matching.**

Definition 1 (Positive Assortative Matching). *Borrowers match with their own type and thus the groups are homogenous in their composition.*

Definition 2 (Negative Assortative Matching). *Borrowers match with other type and thus the groups is heterogenous in its composition.*

With positive assortative matching, the groups would either have both safe types or both risky types. With negative assortative matching each group would have one safe type and one risky type.

Proposition 1 (Positive Assortative Matching). *Joint Liability contracts of the kind described above lead to positive assortative matching.*

To see this, lets examine the process of matching more closely. It is evident that due to the joint liability payment c, everyone want the safest partner they can get. The safer the partner, the lower the probability of incurring the joint liability payment c due to her failure. We need to examine the benefits accruing to the risky type by taking on a safe peer and the loss incurred by the safe type by taking on a risky peer.

$$U_{rs}(r,c) - U_{rr}(r,c) = p_r(p_s - p_r)c \tag{10}$$
$$U_{ss}(r,c) - U_{sr}(r,c) = p_s(p_s - p_r)c \tag{11}$$
$$p_s(p_s - p_r)c > p_r(p_s - p_r)c \tag{12}$$

(10) gives us the gain accruing to the risky type from pairing up with a safe type in stead of a risky type. (11) gives us the loss incurred by a safe type from pairing up with a risky type in stead of another safe type. (12) compares the two equation above and finds that (10) is smaller than (11). It follows that

$$U_{ss}(r,c) - U_{sr}(r,c) > U_{rs}(r,c) - U_{rr}(r,c). \tag{13}$$

Turns out, the safe type's loss exceeds the risky type's gain. The risky type would not be able to bribe the safe type to pair up with her. Joint liability contract leads to positive assortative matching whereby a safe type pairs up with another safe type and the risky type pairs up with another risky type.

Proposition 2 (Socially Optimal Matching)**.** *Positive assortative matching maximises the aggregate expected payoffs of borrowers over all possible matches*

$$U_{ss}(r,c) + U_{rr}(r,c) > U_{rs}(r,c) + U_{sr}(r,c) \tag{14}$$

(14) is obtained by rearranging (13). This implies that positive assortative matching maximises the aggregate expected payoff of all borrowers over different matches.

4.1.1. *Advanced References.* The matching process is determined by the *supermodularity* property of the function that determines the matching process. Becker (1973) discusses how the matching takes place in the marriage market. Topkis (1998) has a comprehensive mathematical treatment of supermodularity. Milgrom and Roberts (1990) and Vives (1990) for explore useful applications in game theory and economics.

4.2. **Indifference Curves.** The indifference curve of borrower type i is given by

$$U_{ij}(r,c) = p_i(x_i - r) - p_i(1 - p_j)c = \bar{k}$$

$$\left[\frac{dc}{dr}\right]_{U_{ii}=\text{constant}} = -\frac{1}{1-p_i}$$

This implies that the safe type's indifference curve is steeper than the risky type's indifference curve.

$$\left|-\frac{1}{1-p_s}\right| > \left|-\frac{1}{1-p_r}\right|$$

This is because the safe type is less concerned about the the joint liability payment c because she is paired up with a safe type. She would like to get a low interest rate r and would happily trade of a higher joint liability payment c in exchange. Conversely, the risky type dislikes the joint liability payment comparatively more. The risky type is stuck with a risky type borrower and incurs the joint liability payment more often than the safe type. She would prefer to have a lower joint liability payment down and does not mind the resulting increase in interest rate. The lender can use the fact that the safe groups and the risky groups trade off the joint liability payment and interest rate payment at different rates to distinguish between the two types of group.

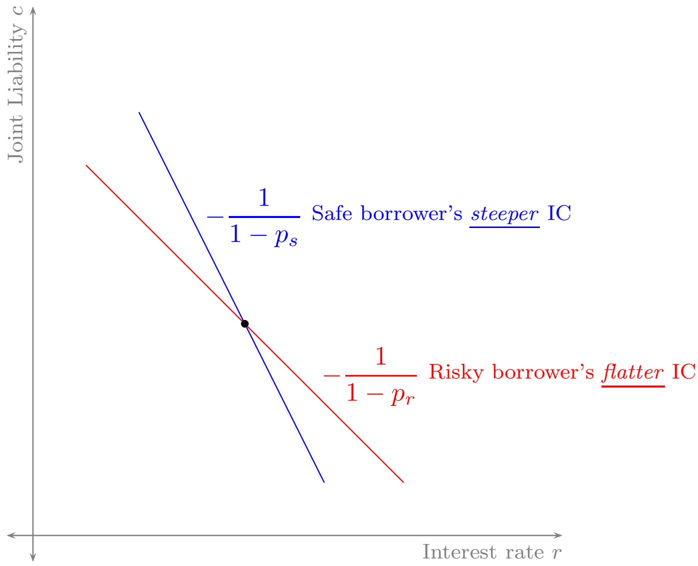

FIGURE 3. caption

FIGURE 4. Risky and Safe Types' Indifference Curves

4.3. **The Lender's Problem.** Now that there are two instruments in the contract, namely r and c, the lender can use the fact the two types trade off r with c at a different rate to induce them to self select into contracts meant for them. The lender offers contracts (r_r, c_r) and (r_s, c_s) and designs the contracts in such a way that the risky type borrowers take up the former and safe type take up the latter contract. The lender offers group contracts (r_r, c_r) and (r_s, c_s) that maximises the borrowers payoff subject to the following constraint:

$$r_r p_r + c_r(1-p_r)p_r \geqslant \rho \quad \Rightarrow \quad \frac{dc}{dr} = -\frac{1}{1-p_r} \tag{L-ZPC$_r$}$$

$$r_s p_s + c_s(1-p_s)p_s \geqslant \rho \quad \Rightarrow \quad \frac{dc}{dr} = -\frac{1}{1-p_s} \tag{L-ZPC$_s$}$$

$$U_{ii}(r_i, c_i) \geqslant \bar{u}, \qquad i = r, s \tag{PC$_i$}$$

$$x_i \geqslant r_i + c_i \qquad i = r, s \tag{LLC$_i$}$$

$$U_{rr}(r_r, c_r) \geqslant U_{rr}(r_s, c_s) \tag{ICC$_{rr}$}$$

$$U_{ss}(r_s, c_s) \geqslant U_{ss}(r_r, c_r) \tag{ICC$_{ss}$}$$

L-ZPC$_i$ is the lender's zero profit condition for borrower type i, PC$_i$ the Participation Constraint for type i, LLC$_i$ the limited liability constraint for type i and ICC$_{ii}$ the incentive compatibility constraint for group (i, i).

To discuss the optimal contract that allows the lender to separate the types, we need to define the $(\hat{r}, \hat{c})$. This is at the point where (L-ZPC$_s$) and (L-ZPC$_r$) cross.

$$\hat{r} = \left(\frac{p_s + p_r - 1}{p_s p_r}\right)\rho$$

$$\hat{c} = \left(\frac{1}{p_s p_r}\right)\rho$$

4.3.1. *Separating Equilibrium in Group Lending.*

Proposition 3 (Separating Equilibrium). *For any joint liability contract* (r, c)

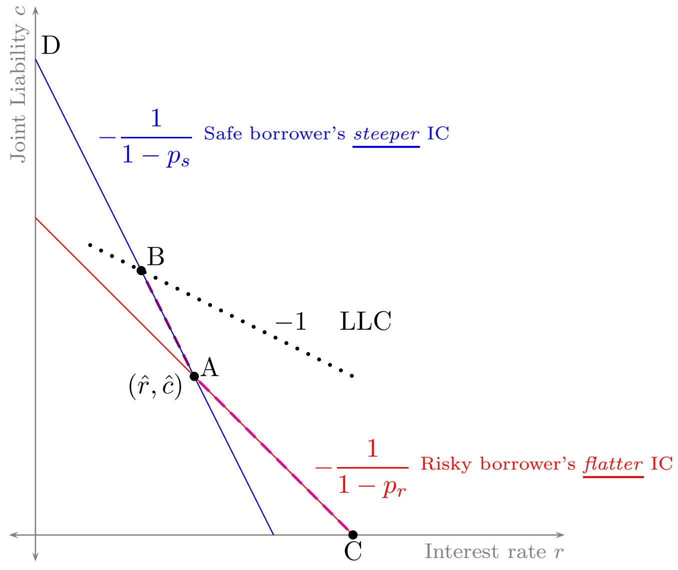

FIGURE 5. Separating Joint Liability Contract

i. *if* $r_s < \hat{r}$, $c_s > \hat{c}$, *then* $U_{ss}(r_s, c_s) > U_{rr}(r_s, c_s)$
ii. *if* $r_r > \hat{r}$, $c_r < \hat{c}$, *then* $U_{rr}(r_r, c_r) > U_{ss}(r_r, c_r)$

The *safe groups* prefer joint liability payment *higher* than $\hat{c}$ and interest rates *lower* than $\hat{r}$. Conversely, the *risky groups* prefer joint liability payments lower than $\hat{c}$ and interest rate higher than $\hat{r}$. With joint liability payment, the lender is able to charge each type a different interest rate. The lender can tailor his contract for the borrower depending on her type. This allows the lender to get back to the first best world where each type was charged a different interest rate.

4.4. **Optimal Contracts.** There are potentially two types of optimal contract. The separating contracts is where the safe group's contract is north-east of $(\hat{c},\hat{r})$ on the safe types' indifference curve and the risky group's contract which is southeast of the this point on the risky types' indifference curve. The second kind of contract is the pooling contract at $(\hat{c},\hat{r})$.

4.5. **Solving the Under-investment Problem.** Under-investment takes place in the individual lending when

$$\rho + \bar{u} < \hat{x} < \frac{p_r}{\bar{p}}\rho + \bar{u}.$$

The safe type are not lent to even though their projects are socially productive. With joint liability separating contracts (above), the safe type are lent to if the project satisfies the limited liability constraint, which leads to the following condition:

$$\hat{x} > \left(\frac{p_s + p_r}{p_r}\right)\rho$$

This condition just ensures that the LLC is to the right of $(\hat{r},\hat{c})$, that is $\hat{x} \geqslant \hat{c} + \hat{r}$. With the pooling contracts explained above, the safe type are lent to if the following condition is met:

$$\hat{x} > \left(\frac{p_s}{\bar{p}}\right)\rho + \beta\bar{u}$$

$$\text{where } \beta \equiv \theta p_r^2 + (1-\theta)p_s^2.$$

This condition insures the participation constraint for the safe type is satisfied.

4.6. **Solving the Over-investment Problem.** Over-investment takes place in the individual lending when

$$\rho + \bar{u} > p_r\bar{x} > \left(\frac{p_r}{\bar{p}}\right)\rho + \bar{u}.$$

The risky type are lent to even though their projects are socially unproductive. In group lending, the risky types participation constraint when she is paired up with another risky type would be given by:

$$p_r\bar{x} - [p_r r + p_r(1-p_r)c] \geqslant \bar{u} \tag{PC$_r$}$$

The lender's zero profit constraint for the risky groups is given by

$$p_r r + p_r(1-p_r)c = \rho$$

This implies that the risky type's participation constraint would be satisfied if

$$p_r\bar{x} \geqslant \rho + \bar{u}$$

This eliminates the over-investment problem. The risky borrowers with the socially unproductive projects will drop out on their own. The condition below ensures that $(\hat{c}, \hat{r})$ satisfies the limited liability constraint.

$$\bar{x} > \left(\frac{1}{p_s} + \frac{1}{p_r}\right)\rho$$

Summary

We have been able to show that the joint liability contract lead to positive assortative matching within groups. Once this matching process takes place, the lender is able to distinguish between the groups of two types using the contract variables r and c. We have also been able to show that this solves the under-investment and over-investment problems prevalent in the individual loan contracts and achieve the first best.

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