Problems

1. Each wealth-less agent has a project which requires an initial investment of £200. The project produces output valued at £500 if it succeeds and £0 when it fails.

   There are two types of agents. For type a agent, the project succeeds with probability 0.2 and fails with probability 0.8. For type b agent, the project succeeds with probability 0.8 and fails with probability 0.2.

   The lender lends to groups of two with a group lending contract as follows: Each agent in the group repays £300 when both her own and her peer's project succeed, £400 when her own project succeeds but her peer's project fails and £0 when her own project fails.

   (a) Show that the type b agent prefers to group with another type b agent as compared to type a agent.

   (b) Explain why type a agent is not able to group with type b agent even though she would like to.

2. Each borrower has a project which requires an investment of 1 unit of capital. With probability π^i the project succeeds and produces output x and with probability $1 - \pi^i$, it fails and produces 0.

   When the agent exerts high effort, the project succeeds with probability $\pi^i = \pi^h$. Conversely, when the agent exerts low effort, the project succeeds with probability $\pi^i = \pi^l$ and the agents obtains a private benefit of value B. ($\pi^l < \pi^h$) The borrowers have no wealth and no alternative source of income and the lender's opportunity cost of capital is ρ. We assume that the lender has all the bargaining strength and extracts all the surplus from the borrower.

   Let the borrower's payoff in individual lending be b_I if her project succeeds and 0 if it fails. Alternatively, the lender may lend to groups of 2. Let each borrower's payoff in group lending be b_G if both group members' projects succeed and 0 otherwise (if one or more member's project fails).

   (a) Write down the lender's problem in individual lending and group lending and find the optimal b_I and b_G.

   (b) Show that the economic rents a borrower obtains in individual lending is higher as compared to the rent she would obtain in group lending.

   (c) Find the least productive project financed in individual and group lending.

   (d) Would the lending efficiency of group lending increase if we reduced the relative bargaining strength of the lender?

3. The question based on Ghatak and Guinnane (1999). In the enforcement model in Section 2.4 (Pages 209-211), the individual liability borrowing repayment condition is

$$u(x) - u(x - r) \leqslant B$$

where x is the output realisation of the project, r is the interest rate due and B is the net present discounted value of having continued access to credit in the future. The joint-liability group lending repayment condition is

$$u(x) - u(x - 2r) \leqslant B.$$

Assume that $B > 0$ and the utility function is logarithmic (to the base e), i.e., $u(x) = \log_e(x)$.

(a) Show that under both types of lending arrangements, borrowers repay only if the output exceeds a certain threshold level. Which type of lending arrangement has a higher threshold?

(b) Find the output range over which group lending does better in terms of repayment than individual lending and vice versa.

(c) Explain why the repayment rate improves if the group members are able to impose social sanctions on each other.

(d) If the group maximises joint welfare in this model (as would be the case if the repayment decisions are taken co-operatively), argue that repayment rates under joint liability will be identical to repayment rates under individual liability.

References

Ghatak, M. and Guinnane, T. W. (1999). The economics of lending with joint liability: theory and practice. *Journal of Development Economics*, 60(1):195–228.