1. (a) Show that Euler's Theorem holds for a *constant returns to scale (CRTS)* production function $F(x_1, x_2)$ with two factors of production x_1 and x_2.

$$F(\lambda x_1, \lambda x_2) = \lambda F(x_1, x_2) \qquad \text{(CRTS)}$$

$$\frac{\partial F}{\partial(\lambda x_1)}\frac{\partial(\lambda x_1)}{\lambda} + \frac{\partial F}{\partial(\lambda x_2)}\frac{\partial(\lambda x_2)}{\lambda} = F(x_1, x_2) \qquad \text{(Diff. wrt } \lambda\text{)}$$

$$F_{x_1}x_1 + F_{x_2}x_2 = F(x_1, x_2)$$

   (b) Interpret the results keeping in mind that the factors are paid their marginal products.

   The output produced, $F(x_1, x_2)$, is exhausted by the total factor payments $F_{x_1}x_1$ and $F_{x_1}x_1$. (Recall that each factor x_i is paid F_{x_i} per unit of the factor used in the production process.)

2. Show that the Euler's theorem hold for a Cobb-Douglas production function $Y = F(x_1, x_2) = (x_1)^{\frac{1}{4}}(x_2)^{\frac{3}{4}}$. *Hint: You have to show that* $F_{x_1}x_1 + F_{x_2}x_2 = F(x_1, x_2)$

$$F_{x_1} = \frac{1}{4}(x_1)^{-\frac{3}{4}}(x_2)^{\frac{3}{4}}, \quad F_{x_2} = \frac{3}{4}(x_1)^{\frac{1}{4}}(x_2)^{-\frac{1}{4}}$$

$$\begin{aligned}
F_{x_1}x_1 + F_{x_2}x_2 &= \left(\frac{1}{4}(x_1)^{-\frac{3}{4}}(x_2)^{\frac{3}{4}}\right)x_1 + \left(\frac{3}{4}(x_1)^{\frac{1}{4}}(x_2)^{-\frac{1}{4}}\right)x_2 \\
&= \frac{1}{4}(x_1)^{\frac{1}{4}}(x_2)^{\frac{3}{4}} + \frac{3}{4}(x_1)^{\frac{1}{4}}(x_2)^{\frac{3}{4}} \\
&= (x_1)^{\frac{1}{4}}(x_2)^{\frac{3}{4}} \\
&= F(x_1, x_2)
\end{aligned}$$