## Supervision for Part IIA, Paper 2, Macroeconomic Policy
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1. Suppose a central bank using monetary targeting minimizes the social welfare loss function

$$L = \frac{1}{2}\alpha\left(\pi - \pi^*\right)^2 + \frac{1}{2}\left(y - y^*\right)^2$$

where $\pi$ is inflation, $y$ aggregate output, $\pi^*$ the inflation target, $y^*$ the output target and $\alpha$ a positive parameter. Assume that $y^* = \kappa\bar{y}$, where $\bar{y}$ is the natural rate of output and $\kappa > 1$. The economy is described by the aggregate supply relation

$$y = \bar{y} + \theta\left(\pi - \pi^e\right) + s$$

where $\pi^e$ denotes private sector inflation expectations, $s$ is a white noise aggregate supply shock with variance $\sigma_s^2$, and $\theta$ is a positive parameter. The central bank adjusts the rate of money growth $m$ to control inflation:

$$\pi = m + v$$

where $v$ is a white noise velocity shock with variance $\sigma_v^2$.
At the beginning of the period, the private sector forms its inflation expectations $\pi^e$ using rational expectations. Subsequently, the supply shock $s$ and velocity shock $v$ are observed. Then, the central bank sets its monetary policy instrument $m$, after which inflation $\pi$ and output $y$ are realized.

   (a) Derive the rate of money growth $m$ that the central bank sets for a given level of private sector inflation expectations $\pi^e$. Explain intuitively how $m$ depends on $\pi^*$, $y^*$, $s$ and $v$.

   (b) Derive the level of private sector inflation expectations $\pi^e$, and the outcome for inflation $\pi$ and output $y$. Explain intuitively how $\pi$ and $y$ depend on $\pi^*$, $y^*$, $s$ and $v$.

   (c) Compute the expected value and variance of inflation and output: $E\left[\pi\right]$, $E\left[y\right]$, $Var\left[\pi\right]$ and $Var\left[y\right]$. Give an economic interpretation of the results.

   (d) Suppose the government decides to delegate monetary policy to a new central banker. Explain the effect on the expected value and variance of inflation and output if the new central banker (indicated by CB) has an objective function with

      i. a higher parameter $\alpha$, such that $\alpha_{CB} = 2\alpha$; or
      ii. a lower parameter $\kappa$, such that $\kappa_{CB} = 1$.

      In each case, discuss whether delegation to such a central banker would be desirable.

2. Consider the following macroeconomic model. Aggregate demand is given by:

$$y_t = m_t - p_t + v_t \tag{1}$$

and aggregate supply by:

$$y_t - y_t^p = \beta(p_t - {}_{t-1}\, p_t^e) \tag{2}$$

where $y$ is the log of real output, $y^p$ is the log of trend output, $m$ is the log of the money stock, $p$ is the log of the price level and $v$ is the log of (constant) velocity. The subscript $t$ indicates the time period. ${}_{t-1}p_t^e$ is the expectation formed in period $t-1$ of what the price level will be in period $t$. It is assumed that the policymaker follows a rule for the stock of money of the form

$$m_t = \alpha y_{t-1} + \epsilon_t \tag{3}$$

where $\epsilon_t$ is a zero mean surprise shock to monetary policy.

(a) Suppose expectations are formed adaptively. Write down an expression for how expectations are formed in this case and comment on the properties of this relationship.

(b) There a single positive shock to $\epsilon$ in period $t$. Show diagrammatically how output and the price level return to equilibrium. Make clear what determines the speed with which equilibrium is re-established.

(c) Suppose now expectations are formed rationally so ${}_{t-1}p_t^e = E(p_t \mid I_{t-1})$. Explain what is meant by this.

(d) What is the effect now on output and the price level of a shock to $\epsilon_t$? Show diagrammatically how output and the price level return to equilibrium. Make clear what determines the speed with which equilibrium is re-established.