Sequential Group Lending with Moral Hazard

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Abstract

We examine a microfinance institution's ability to lend to low productivity project undertaken by wealth-less borrowers in two-task moral hazard environment where borrower exert effort on their project and to influence their peer's effort level. We compare the mechanisms of individual, simultaneous and sequential group lending while varying the peer-influence function. We show that the sequential group lending has the smallest lower-bound on project productivity if the cost of influencing peer's action is negligible. Conversely, simultaneous lending has the smallest lower bound if cost of influencing the peer tends to infinity.

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1 Introduction

In its initial flourish, microfinance became synonymous with the simple idea of group lending. The academic literature was able to show that theoretically joint-liability[1] group-lending was more efficient than individual lending.[2] Yet, in the real world, microfinance uses a complex web of mechanisms in practice that are not fully explored in the the academic literature. Inevitably, there has been a backlash supported by empirical evidence that has questioned the advantage simple group lending has over individual lending.[3]

The idea of group lending may suggest that by default all members of the group obtain their loans simultaneously from the lender. Yet, the mechanism of sequential lending, where loans are disbursed sequentially within the group, is widely used in practice[4] and is one of those aforementioned complex web of mechanisms that empowers group lending.[5]

In our model, each wealth-less borrower has two distinct costly tasks, exerting effort on their own project and exerting influence on their peer's effort. Specifically, we assume that effort is binary, peer-influence is a continuous variable and there are positive cross-complementarities between the two tasks, i.e., influence by peer reduces the opportunity cost of high effort

[1]With joint-liability, a lender makes a borrower is made liable for their peer's failure.

[2]Morduch (1999), Ghatak and Guinnane (1999) and Karlan and Morduch (2010) are excellent summaries of the theoretical literature on group lending.

[3]Karlan and Morduch (2010), Giné and Karlan (2009)

[4]Grameen Bank used sequential lending initially for two decades and Self Help Groups in India and Grameen replicators continue to use it today. For detailed description of the lending mechanisms, see Aniket (2003) and Armendáriz de Aghion and Morduch (2005, Pages 87-88) for Grameen Bank and Harper (2002) and Aniket (2006) for Self Help Groups. ROSCAs also involve sequential allocation of credit amongst the members. See Besley et al. (1993) and (Klonner, 2008).

[5]Varian (1990) and Chowdhury (2005) have previously modelled sequential lending and this paper complements their analysis.

for a borrower. The costly peer-influence variable captures the connection amongst the borrowers and is reflective of the environment they live in. The objective of this paper is to compare the efficiency of sequential group lending with simultaneous group lending and individual lending, while varying this particular connection between the borrowers. Stiglitz (1990) and Ghatak and Guinnane (1999) have shown previously that joint-liability group contracts are more efficient than individual lending contracts in a single-task moral hazard environment, assuming that peer-influence is costless. Thus, the environment the borrowers live in does not matter in their papers.

The metric we choose to compare the various lending mechanisms is the lower-bound on project productivity.[6] There are apocryphal stories of poor possessing extremely high productivity projects. In reality, there may be considerable variation in productivity of the projects the poor possess (Banerjee and Duflo, 2007).[7] Low project productivity maybe be as important a factor as collateral in restricting credit to the poor.[8] Reducing the project *productivity lower-bound* is imperative if entrepreneurial activity amongst the poor is to be facilitated.

[6]By facilitating a project, a loan contract creates a potential surplus. How that surplus is shared between the lender and the borrower(s) depends on their relative bargaining strength, which in turn is largely determined by the market structure. Irrespective of the market structure, the least productive projects financed has no surplus left. Thus, as well as being relevant in the context of microfinance, the metric of least productive project financed is a good proxy for the efficiency of a lending mechanism.

[7]Low productivity for poor entrepreneurs could be a result of either inherent attributes like lack of skills and ill-health or exogenous factors like lack of public good and markets access. If microfinance has a high lower-bound for project productivity, it would not able to reach either individual or areas where the aforementioned factors conspire to keep project productivities low.

[8]Field experiment conducted by Field et al. (2013) looks at what discourages high-return illiquid investment and finds that it is early repayment required by microfinance institutions.

The paper shows that the optimal output-contingent contract in the simultaneous group lending is an *extreme joint-liability* contract with no induced peer-influence.[9] With no peer-influence, simultaneous lending is functionally very similar to individual lending.[10] Though comparatively, the borrowers get lower expected payoffs in simultaneous lending because positive payoffs occur less often with the extreme joint liability contract.

In sequential lending, a randomly chosen borrower borrows first. The second borrower gets the loan with certainty if the first borrower succeeds and with a pre-specified probability if the first borrower fails. The paper shows that with sequential lending, the lender induces positive peer-influence and some joint-liability is optimal in the contract but extreme joint-liability is not optimal.[11] This is because with extreme joint liability, the second borrower will have no incentive to pursue the second project if the first project fails. In a result relevant for our specific metric, we show that for projects in the vicinity of the productivity lower-bound, the second borrower should always be denied the loan if the first borrower fails.[12] This is not true for projects with higher productivity.

We vary the effectiveness of peer influences and look for the mechanism that yields the smallest project *productivity lower-bound*. The main result of

[9] An extreme joint liability contract is an all or nothing contract, where the borrower get positive payoff only if both borrowers succeed and zero (due to limited liability) otherwise.

[10] Giné and Karlan (2009) find that (simultaneous) group and individual lending have very similar default rates. The lack of peer-influence when influencing the peer is costly may explain this.

[11] If the borrowers succeed and their peer fails, they are penalised for their peer's failure but still obtain positive payoffs.

[12] This is because at the margin, the additional output from the second borrower's projects is less than the additional borrower payoff required to continue lending after the first borrower has failed.

the paper is that if the cost of reducing peer's private benefit is sufficiently low, sequential lending has the smallest productivity lower-bound amongst the three lending mechanisms. Further, sequential lending approaches first-best productivity lower-bound as cost of reducing peer's private benefit tends to zero. Conversely, if the cost of influencing peer's action is sufficiently high, simultaneous lending has the smallest productivity lower-bound. Thus, sequential lending may be more appropriate for an intimate rural setting and simultaneous lending more appropriate for an urban ghetto.

Sequential lending has previously been modelled by Varian (1990)[13] in an adverse selection and by Chowdhury (2005) in a costly state verification or auditing environment.[14] In Chowdhury (2005), the auditor invests in capacity that increases the probability of finding the project output. Chowdhury (2005) shows that sequential lending generates positive peer-auditing where as simultaneous lending fails to generate any.

This paper is different from Chowdhury (2005) in the following ways. Chowdhury (2005) analyses the lending mechanisms in a single-task environment whereas this paper does so in a two-task environment. This paper explicitly derives the optimal contract where as Chowdhury (2005) assumes an extreme joint-liability contract where the government sets the loan interest rate. By explicitly deriving the optimal contract, we are able to show that the extent of optimal joint-liability varies between simultaneous and sequential

[13]With two types of borrowers, Varian (1990) shows that if the high productivity and the low productivity type are grouped together, sequential lending gives the high productivity type the incentive to school the low productivity type, thus raising the overall productivity of the group.

[14]Cason et al. (2012) test the theoretical model of Chowdhury (2005) in the experimental laboratory setting and finds evidence in support.

lending. Further, whereas Chowdhury (2005) evaluates all mechanisms in the same environment, by varying the connection between borrowers we are able to explore how the optimal lending mechanism varies with it. Chowdhury (2005) assumes that in sequential lending, if the first borrower fails, second borrower is denied the loan with certainty. We show that there is caveat to this, i.e., this is not optimal for low-productivity projects in the vicinity of the productivity lower-bound.

As compared to the wider literature in microfinance,[15] what is distinctive about this paper is that it explicitly derives the optimal contract and show the extent to which joint liability is optimal for each mechanism.[16] This is also the first paper to try to explicitly vary the connection between the borrowers and show that different mechanisms may be appropriate in different environments. It naturally follows that the external validity of empirical studies in microfinance need careful consideration. A mechanism may be very effective in one environment and less so in another one. Thus, empirical studies should consider the efficacy of mechanisms in microfinance under a wide variety of environments before drawing any definitive conclusions. Further, before we summarily discard decades of microfinance expertise in group-lending, we should carefully evaluate, both theoretically and empirically, the complex web of mechanisms it employs in practice.

[15] Joint-liability contracts have been assumed in papers like Stiglitz (1990), Ghatak and Guinnane (1999), Chowdhury (2005), Besley and Coate (1995), Conning (2000), Van Tassel (1999)

[16] Following this approach, we show that whereas extreme joint-liability is optimal for simultaneous lending, only some joint-liability it optimal for sequential lending.

2 Environment

A project requires a lump-sum investment of 1 unit of capital and produces an uncertain and observable outcome x, valued at $\bar{x} \in \{0, \infty\}$ when it succeeds (s) and 0 when it fails (f). Each agent has access to only one specific project. We will use $\bar{x}$, its success value, as a mnemonic for both the project and the agent who has access to that project. The population of agents is distributed over the project range $\bar{x} \in [0, \infty)$.

The agents are risk neutral, with zero reservation wage and no wealth. Agents may choose to pursue the aforementioned project with a high (H) or low (L) effort e, which is unobservable. With a high (low) effort, $\bar{x}$ is realised with a probability $\bar{\pi}$ $(\underline{\pi})$ and 0 with $1 - \bar{\pi}$ $(1 - \underline{\pi})$. $(\bar{\pi} > \underline{\pi} > 0)$

By exerting low effort, agents obtain private benefits of value B from the project which are non-pecuniary and non-transferable amongst the agents. Private benefits can be curtailed by peer-influence c. An agent can generate peer-influence c by bearing non-pecuniary cost c. e and c are observable amongst the agents but not to the lender. We impose the following assumption on the peer-influence function $B(c)$.

Assumption 1 (Peer-Influence Function $B(c)$)**.** *$B(c)$ is continuous and at least once differentiable $\forall\ c \geqslant 0$. $B(c) \geqslant 0$, $B'(c) \leqslant 0$, $\forall\ c \geqslant 0$, $B(0) = B_0 > 0$ and $\lim_{c \to \infty} B(c) = 0$. $B^{-1}(\,\cdot\,)$ is defined as the the inverse function of $B(c)$.*

The lender is a risk-neutral profit-maximising monopolist in the loan market with access to capital at cost ρ. The lender can observe the initial capital invested, the project output and fully enforce contracts.[17]. The lender cannot directly influence private benefits himself and can only incentivise the agents through pecuniary payoffs. We also assume that there is full commitment to the loan contract from the lender and borrowers side. To focus on the moral hazard problem, we assume that $E[x \mid H] - \rho \geqslant 0 \geqslant E[x \mid L] - \rho + B(0)$, i.e., from a social perspective, high effort on a project breaks-even but low effort does not break-even.

Each agent borrows 1 unit of capital to undertake their project. In individual lending, a borrower's payoff b_i is contingent on $i = \{s, f\}$. In group lending, the borrower's payoff b_{ij} is contingent on borrower's output $i = \{s, f\}$ and her peer's output $j = \{s, f\}$. We assume that borrowers' project outcomes in a group is statistically independent.[18] Assumption 2 ensures that the payoffs are always non-negative.

Assumption 2 (Limited Liability)**.** *In individual lending,* $b_i \geqslant 0 \vee i = \{s, f\}$*. In group lending,* $b_{ij} \geqslant 0 \vee i, j = \{s, f\}$*.*

3 Individual Lending

In individual lending, the borrower undertakes a project if she accepts the lender's contract. With perfect information, the lender can observe the borrower's effort level and project outcome. The lender offers the borrower a

[17]To focus on the hidden action problem, we assume away the problems of hidden type, costly state verification and contract enforcement. These problems are explored comprehensively in papers like Ghatak (1999), Ghatak (2000), Ghatak and Guinnane (1999), Rai and Sjöström (2004) and Besley and Coate (1995).

[18]That is, if agent B_1 exerts high effort and agent B_2 exerts low effort in group lending, the likelihood of state ss is simply $\underline{\pi}\overline{\pi}$.

state-contingent contract b_i that minimises the borrower's payoff $E[b_i \mid H]$ subject to borrower's participation constraint $E\left[b_i \mid H\right] \geqslant 0$ and the limited liability constraint $b_s, b_f \geqslant 0$. The optimal contract stipulates that the borrower exerts high effort and $b_s = b_f = 0$. The lender's break-even condition $E[x \mid H] - E[b_i \mid H] - \rho \geqslant 0$ is satisfied for $\bar{x} \geqslant \frac{\rho}{\bar{\pi}}$.

3.1 Second-Best

With incomplete information, the borrower's effort is unobservable to the lender. To elicit high effort, the lender would have to satisfy the borrower's incentive compatibility condition $E[b_i \mid H] \geqslant E[b_i \mid L] + B_0$.[19] The lender would offer the borrower a contract where $b_s = \frac{B_0}{\Delta\pi}$ and $b_f = 0$ where $\Delta\pi = \bar{\pi} - \underline{\pi}$.[20] The lender would break even for projects $\bar{x} \geqslant \frac{\rho}{\bar{\pi}} + \frac{B_0}{\Delta\pi} = \bar{x}_{indv}$. Conversely, the lender could elicit low effort by offering a contract $b_s = b_f = 0$ and break even for projects $\bar{x} \geqslant \frac{\rho}{\underline{\pi}}$. We henceforth assume that $\rho \geqslant \frac{\bar{\pi}\underline{\pi}}{\Delta\pi}\left[\frac{B_0}{\Delta\pi}\right]$, which follows from $\frac{\rho}{\underline{\pi}} \geqslant \bar{x}_{indv}$. With this assumption, the high effort contract yields a smaller productivity lower-bound than the low effort contract.

4 Group Lending

A groups consists of two borrowers, B_1 and B_2, seeking loans from the lender to undertake their respective projects. The lender can observe the state ij, where $i = \{s, f\}$ and $j = \{s, f\}$ are the B_1's and B_2's project outcome respec-

[19] If the incentive compatibility and the limited liability constraints are satisfied, the participation constraint would always be satisfied.

[20] The contract ensures that the incentive compatibility constraint binds and the limited liability constraint binds only for state f.

tively,[21] and offers the both borrowers a symmetric group-lending contract (b_{ij}). The borrowers borrow simultaneously in section 4.1 and sequentially in section 4.2. We derive the optimal contract and in the process determine the extent of joint-liability that is optimal for each group lending mechanism. Confining our analysis to symmetric contacts in group lending allows us to pin down the effect of sequencing the loan. Further, given that the borrowers usually get symmetric contracts in microfinance, it is not unreasonable to assume so in our analysis here.

4.1 Simultaneous Group Lending

In simultaneous group lending, borrowers borrow simultaneously. The timing of the game is as follows: $t = 0$: The lender offers B_1 and B_2 an identical contract $(b_{ss}, b_{sf}, b_{fs}, b_{ff})$. If they accept the contract, the game continues. Otherwise, it terminates. $t = 1$: B_1 and B_2 choose their respective peer-influence intensities $c_1 \in [0, \infty)$ and $c_2 \in [0, \infty)$ simultaneously. $t = 2$: Given (c_1, c_2) chosen at $t = 1$, B_1 and B_2 choose their respective effort levels $e_1 \in \{H, L\}$ and $e_2 \in \{H, L\}$ simultaneously. $t = 3$: B_1 and B_2's project outcome is realised. Both borrowers get payoffs b_{ij} depending on the realised state ij, where $i, j = \{s, f\}$. Lemma 1 summarises the conditions under which the both borrowers have the incentive to exert high effort.[22]

[21] ss where both B_1 and B_2's project succeeds, ff where both B_1 and B_2's project fails, sf where B_1's project succeeds and B_2's project fails and fs where B_1's project fails and B_2's project succeeds.

[22] If borrower $k = \{1, 2\}$ exerts effort e_k, $P(i \,|\, e_k)$ is the probability of a borrower k's project resulting in outcome i. $E[b_{ij} \,|\, e_1 e_2] = \sum_i \sum_j P(i \,|\, e_1) P(j \,|\, e_2)\, b_{ij}$. For ease of exposition, we use the mnemonic $P(s \,|\, e_k) = \pi_k$ where $\pi_k = \bar{\pi}$ if $e_k = H$ and $\pi_k = \underline{\pi}$ if $e_k = L$.

Lemma 1. *Both borrowers exert high effort on their respective projects if the lender's contract* $(b_{ss}, b_{sf}, b_{fs}, b_{ff})$ *satisfies the following conditions.*

$$E(b_{ij} \mid HH) - E(b_{ij} \mid LL) \geqslant B_0 \tag{1}$$

$$E(b_{ij} \mid HH) - E(b_{ij} \mid LH) \geqslant B_0 \tag{2}$$

We show in Appendix A that if (1) and (2) are satisfied, the borrowers would exert high effort on their own projects at $t = 2$. This is true for all possible (c_1, c_2) combination chosen at $t = 2$ where $c_1, c_2 \in [0, \infty)$.

(2) is B_1's (and symmetrically B_2's) incentive compatibility condition associated with effort level. For a given (c_1, c_2) combination if B_2 (B_1) exerts high effort, this condition ensures that B_1 (B_2) is no worse off exerting high effort as compared to low effort. (1) is the group incentive compatibility condition which ensures that both borrowers prefer exerting high effort over both exerting low effort. Lemma 1 implies that $c_1 = c_2 = 0$. In both (1) and (2), the borrower are compensated for forgoing B_0, the maximal value of private benefits, because a borrower i's peer can always choose to not influence their peer by opting for $c_j = 0$ at $t = 1$.

The lender's problem ($\mathrm{P_{sim}}$) is $\min_{b_{ij}} E[b_{ij} \mid HH]$ subject to (1) and (2). The problem is solved in Appendix A.1 and the results are summarised in Proposition 1.

Proposition 1. *In simultaneous group lending, the optimal contract has the following characteristics:*

i. (1) *binds and* (2) *is slack,*

ii. there is no peer-influence, i.e., $c_1 = c_2 = 0$ *and*

The optimal contract has extreme joint-liability, i.e., $b_{ss} = \frac{B(0)}{\bar{\pi}^2 - \underline{\pi}^2} > 0,\ b_{sf} = b_{fs} = b_{ff} = 0.$

We show in Appendix A.1 that (1) binds and (2) is slack in the optimal contract. Further, we show that that the Lagrangian associated with the problem ($\mathrm{P}_{\mathrm{sim}}$) is globally decreasing in both b_{sf}, b_{fs} and b_{ff} at the solution. Given Assumption 2, it is optimal to set $b_{sf} = b_{fs} = b_{ff} = 0$.[23] A binding (1) gives us $b_{ss} = \frac{B(0)}{\bar{\pi}^2 - \underline{\pi}^2}$. Thus, extreme joint-liability contracts, where the borrowers get nothing if either borrower fails, is optimal in simultaneous group lending.

If peer-influence is costly, simultaneous group lending is not very different from individual lending given that $c_1 = c_2 = 0$. The borrower's expected payoff is lower in simultaneous group lending simply because borrowers get paid with a lower probability due to the nature of extreme joint liability contract.[24]

4.2 Sequential Group Lending

In sequential group lending, only one borrower in the group can borrow at a time. We assume that the lender randomly chooses the first borrower in the group. Let's call the first borrower B_1. If B_1's project fails, B_2 gets a loan

[23] For the lender, state ss is more informative than the states sf, fs and ff about the borrowers' respective effort levels. Concentrating the payoff in ss allows the lender to give the borrowers the requisite incentive to exert high effort at the lowest possible cost in terms of expected payoffs (Hölmstrom, 1979). Thus, the proof in Appendix A.1 confirms the intuitive proof about extreme joint liability set out in Conning (2000, page 17).

[24] $\bar{\pi}\frac{B_0}{\Delta\pi}$ and $\bar{\pi}^2\frac{B_0}{\bar{\pi}^2 - \underline{\pi}^2}$ are the borrower's expected payoffs in individual and simultaneous group lending.

with a probability $\varepsilon \in [0, 1]$. Conversely, if B_1's project succeeds, B_2 gets the loan with certainty.

In sequential group lending, we have an additional state of the world f.[25] f occurs when B_1 fails and B_2 does not get the loan. b_f is the borrowers' payoff in state f. The timing of the game is as follows:

t=0: The lender offers B_1 and B_2 an identical contract $(b_{ss}, b_{sf}, b_{fs}, b_{ff}, b_f)$. If they accept the contract, the game continues. Otherwise, it terminates. t=1: B_2 chooses peer-influence intensity c_2.[26] t=2: B_1 chooses her effort level e_1. t=3: B_1's project outcome is realised. If B_1's project fails, with probability $(1-\varepsilon)$ the game terminates and both borrowers get payoff b_f. With probability ε, the game continues. Conversely, if B_1's project succeeds, the game continues with certainty. t=4: B_1 chooses peer-influence intensity c_1^s if B_1's project has succeeded and c_1^f if it has failed. t=5: B_2 chooses effort level e_2^s if the B_1's project has succeeded and e_2^f if it has failed. t=6: B_2's project outcome is realised. Both borrowers get payoff b_{ij} depending on realised state ij, where $i, j = \{s, f\}$.

Lemma 2. *Both borrowers exert high effort on their respective projects if the lender's contract $(b_{ss}, b_{sf}, b_{fs}, b_{ff}, b_f)$ satisfies the following conditions.*

$$b_{ss} - b_{sf} \geqslant \frac{1}{\Delta\pi} \max\left[B(c_1^s), c_1^s\right] \quad (3)$$

$$b_{fs} - b_{ff} \geqslant \frac{1}{\Delta\pi} \max\left[B(c_1^f), c_1^f\right] \quad (4)$$

$$\bar{\pi}(b_{ss} - \varepsilon b_{fs}) + (1-\bar{\pi})(b_{sf} - \varepsilon b_{ff}) - (1-\varepsilon) b_f \geqslant \frac{1}{\Delta\pi} \max\left[B(c_2), c_2\right] \quad (5)$$

[25] We add this new state f to the states ij, where $i, j = \{s, f\}$

[26] This is the intensity with which B_2 chooses to influence her peer B_1.

We show in Appendix B that if (3), (4) and (5) are satisfied, B_1 and B_2 will exert high effort at $t = 2$ and $t = 5$ respectively. (5) ensures that B_2 has the requisite incentive to choose peer-influence intensity of at least $c''' = B^{-1}\big[\Delta\pi[\bar{\pi}(b_{ss} - \varepsilon b_{fs}) + (1-\bar{\pi})(b_{sf} - \varepsilon b_{ff}) - (1-\varepsilon)b_f]\big]$ at $t = 1$ such that B_1 will have the incentive to exert high effort at $t = 2$. At $t = 3$, the success and failure of B_1's project creates two distinct subgames. (3) and (4) are the respective incentive conditions associated with the subgames that occur after B_1's project succeeds and fails. (3) and (4) ensure that B_1 has the incentive to choose peer-influence intensity of at least $c_1^{s\prime} = B^{-1}\left[\Delta\pi(b_{ss} - b_{sf})\right]$ and $c_1^{f\prime\prime} = B^{-1}\left[\Delta\pi(b_{fs} - b_{ff})\right]$ at $t = 4$ such that B_2 will have the incentive to exert high effort at $t = 5$ in the respective subgames.

The lender's problem ($\mathrm{P_{seq}}$) is $\min_{b_{ij}} E[b_{ij} \mid HH]$ subject to (3), (4) and (5). The problem is solved in Appendix B.1 and the results are summarised in Proposition 2.

Proposition 2. *In sequential group lending, the optimal contract has the following characteristics:*

i. (3) *remains slack and* (4) *and* (5) *bind,*

ii. $c_1^s \in \left[B^{-1}\left(\Delta\pi(b_{ss} - b_{sf})\right), c_{seq}\right]$, $c_2 = c_1^f = c_{seq}$ *where* $c_{seq} = B(c_{seq})$.

(P_{seq}) *is solved by a range of contracts where* $b_{ff} = b_f = 0$, $b_{fs} = \frac{c_{seq}}{\Delta\pi}$, $b_{ss} = \left(\frac{1+\bar{\pi}\varepsilon}{\bar{\pi}}\right)\frac{c_{seq}}{\Delta\pi} - \left(\frac{1-\bar{\pi}}{\bar{\pi}}\right)b_{sf}$ *and* $b_{sf} \in \left[0, \bar{\pi}^2\varepsilon\frac{c_{seq}}{\Delta\pi}\right]$. *This range of contracts exhibits joint-liability, but not the extreme form.*

We show in Appendix B.1 that $b_{ff} = b_f = 0$ since the Lagrangian associated with the problem ($\mathrm{P_{seq}}$) is globally decreasing in both b_{ff} and b_f at the solution. A contract that satisfies (5) will always satisfy (3) and may leave it slack. From the binding constraints (4) and (5) we get a contract where $b_{fs} = \frac{c_{seq}}{\Delta\pi}$ and $\bar{\pi} b_{ss} + (1-\bar{\pi}) b_{sf} = (1+\bar{\pi}\varepsilon)\left[\frac{c_{seq}}{\Delta\pi}\right]$. (3) will be satisfied if $b_{ss} - b_{sf} \geqslant \frac{c_{seq}}{\Delta\pi}$. Thus, for a contract to satisfy (3), (4) and (5), the contract has to take a form where $b_{ss} = (1+\bar{\pi}\varepsilon)\left[\frac{c_{seq}}{\Delta\pi}\right] - \left(\frac{1-\bar{\pi}}{\bar{\pi}}\right) b_{sf}$ and $b_{sf} \in \left[0, \bar{\pi}^2\varepsilon\left[\frac{c_{seq}}{\Delta\pi}\right]\right]$.[27] This contract exhibits some joint liability[28] but does not exhibit extreme joint liability that we saw in simultaneous group lending.[29]

Given a particular contract, B_1 can choose $c_1^s \in \left(B^{-1}[\Delta\pi(b_{ss} - b_{sf})], c_{seq}\right)$. As we show in Appendix B, since B_1 chooses c_1^s before B_2 chooses e_2^s, B_1 would inevitably choose $c_1^s = B^{-1}(\Delta\pi(b_{ss} - b_{sf}))$. This would imply that $b_{ss} - b_{sf} \geqslant \frac{c_1^s}{\Delta\pi}$ and $b_{ss} - b_{sf} = \frac{B(c_1^s)}{\Delta\pi}$.

5 Comparing the Lending Mechanisms

The two group lending mechanisms put a lower bound on the group's average productivity and not explicitly on individual borrower's project productivities. Variation in individual project productivities within the group is entirely feasible. Thus, group lending has an added advantage over individual lending, where $\bar{x} \geqslant \bar{x}_{indv}$ for each individual borrower.

Proposition 3. *To borrow in individual lending, the lower bound on project*

[27] There are a continuum of optimal contracts because the trade-off between b_{ss} and b_{sf} is identical in the lender's objective function and constraint (5). The contracts range from $(b_{ss}, b_{sf}, b_{fs}, b_{ff}, b_f)$ that vary from $\left((1+\bar{\pi}\varepsilon)\frac{c_{seq}}{\Delta\pi}, 0, \frac{c_{seq}}{\Delta\pi}, 0, 0\right)$ to $\left((1+\bar{\pi}^2\varepsilon)\frac{c_{seq}}{\Delta\pi}, \bar{\pi}^2\varepsilon\frac{c_{seq}}{\Delta\pi}, \frac{c_{seq}}{\Delta\pi}, 0, 0\right)$.

[28] $b_{ss} - b_{fs} \in \left[\bar{\pi}^2\varepsilon\frac{c_{seq}}{\Delta\pi}, \bar{\pi}\varepsilon\frac{c_{seq}}{\Delta\pi}\right]$, $b_{sf} - b_{ff} \in \left[0, \bar{\pi}^2\varepsilon\frac{c_{seq}}{\Delta\pi}\right]$, $b_{ss} - b_{sf} \in \left[(1+\bar{\pi}\varepsilon)\frac{c_{seq}}{\Delta\pi}, \frac{c_{seq}}{\Delta\pi}\right]$ and $b_{fs} - b_{ff} = \frac{c_{seq}}{\Delta\pi}$.

[29] $b_{fs} > 0, b_{sf} \geqslant 0$.

productivity is $\bar{x}_{indv} = \frac{\rho}{\bar{\pi}} + \frac{B_0}{\Delta\pi}$. *To borrow in simultaneous and sequential group lending, the lower bound on expected average project productivity of the group is* $\bar{x}_{sim} = \frac{\rho}{\bar{\pi}} + \left[\frac{\bar{\pi}}{\bar{\pi}+\underline{\pi}}\right]\frac{B_0}{\Delta\pi}$ *and* $\bar{x}_{seq} = \frac{\rho}{\bar{\pi}} + \frac{2}{(1+\bar{\pi})}\left[\frac{c_{seq}}{\Delta\pi}\right]$ *respectively.*

Substituting the simultaneous and sequential group lending contract into the lender's break even condition $E[x \mid HH] \geqslant \rho + E[b_{ij} \mid HH]$ gives us $\bar{x} \geqslant \bar{x}_{sim} = \frac{\rho}{\bar{\pi}} + \left[\frac{\bar{\pi}}{\bar{\pi}+\underline{\pi}}\right]\frac{B_0}{\Delta\pi}$ and $\bar{x} \geqslant \bar{x}_{seq}(\varepsilon) = \frac{\rho}{\bar{\pi}} + \frac{2[1+\varepsilon]}{(1+\bar{\pi})+(1-\bar{\pi})\varepsilon}\left[\frac{c_{seq}}{\Delta\pi}\right]$ respectively.[30] (*Referees' Appendix C.1*)

$\frac{d\bar{x}_{seq}}{d\varepsilon} = \frac{4\bar{\pi}}{[(1+\bar{\pi})+(1-\bar{\pi})\varepsilon]^2}\left[\frac{c_{seq}}{\Delta\pi}\right] > 0$ shows that $\bar{x}_{seq}$, the lower bound for sequential group lending, is increasing in ε. This is because along the lower bound $\bar{x}_{seq}(\varepsilon)$ locus, the expected marginal cost always overwhelms the expected marginal output from increasing ε. That is, increasing ε at the margin never creates a surplus that could potentially decrease $\bar{x}_{seq}$.[31] Thus, setting $\varepsilon = 0$ minimises the lower bound on group's average productivity and the lender would not continue the game at $t = 3$ if B_1's project fails.

5.1 Varying the Peer-influence Function

To compare the lending mechanisms, we assume a slightly modified peer-influence function $B(c, \beta) = B_0 + \beta \cdot b(c)$, which is separable in B_0 and $b(c)$, the reduction in private benefit from peer-influence.

Assumption 3. *$b(c)$ is continuous and at least once differentiable* $\forall\ c \geqslant 0$. $b(c) \leqslant 0$, $b'(c) \leqslant 0$, $\forall\ c \geqslant 0$. $b(0) = 0$ *and* $\lim_{c\to\infty} b(c) = -B_0$

[30] The lender's expected cost of capital is $[(1+\bar{\pi}) + (1-\bar{\pi})\varepsilon]\,\rho$. The expected output is $\bar{\pi}\,[(1+\bar{\pi}) + (1-\bar{\pi})\varepsilon]\,\bar{x}$ and the expected borrower's payoffs are $2\bar{\pi}[1+\varepsilon]\left[\frac{c_{seq}}{\Delta\pi}\right]$.

[31] $\rho + 2\bar{\pi}b_{fs}$ is the expected marginal cost and $\bar{\pi}\bar{x}$ is the expected marginal output of increasing ε. The lender would increase ε at the margin if $\bar{x} > \frac{\rho}{\bar{\pi}} + 2\left[\frac{c_{seq}}{\Delta\pi}\right]$. Given that $\bar{x}_{seq}(0) = \frac{\rho}{\bar{\pi}} + \frac{2}{(1+\bar{\pi})}\left[\frac{c_{seq}}{\Delta\pi}\right]$ and $\bar{x}_{seq}(1) = \frac{\rho}{\bar{\pi}} + 2\left[\frac{c_{seq}}{\Delta\pi}\right]$, this condition is never satisfied along the lower bound locus $\bar{x}_{seq}(\varepsilon)$ for $\varepsilon \in [0,1]$.

$\beta \in [0, \infty)$ captures the effectiveness of peer-influence in reducing private benefits. As $\beta \to \infty$, an infinitesimal amount of peer-influence drives the private benefits very close to zero. This could represents the situation in the settled rural community where the borrowers are extremely effective in influencing their peer's action. Conversely, as $\beta \to 0$, even an extremely high peer-influence intensity will have no impact on a borrower's private benefit. This may represent the urban ghetto where a borrower's peer-influence may not have any influence at all on her peer's action.

Proposition 4. *There exists $\hat{\beta}_1$ and $\hat{\beta}_2$ such that $\bar{x}_{seq}(\hat{\beta}_1) = \bar{x}_{indv}$ and $\bar{x}_{seq}(\hat{\beta}_2) = \bar{x}_{sim}$. $\bar{x}_{seq}(\beta) > \bar{x}_{indv} > \bar{x}_{sim}\ \forall\ \beta \in (0, \hat{\beta}_1)$, $\bar{x}_{indv} > \bar{x}_{seq}(\beta) > \bar{x}_{sim}\ \forall\ \beta \in (\hat{\beta}_1, \hat{\beta}_2)$ and $\bar{x}_{indv} > \bar{x}_{sim} > \bar{x}_{seq}(\beta)\ \forall\ \beta \in (\hat{\beta}_2, \infty)$.*

$\bar{x}_{indv} > \bar{x}_{sim}$ follows directly from Proposition 3. With the new peer-influence function, we have $\bar{x}_{seq}(\beta) = \frac{\rho}{\bar{\pi}} + \frac{2}{(1+\bar{\pi})}\left[\frac{B(c_{seq}(\beta), \beta)}{\Delta\pi}\right]$ where $B(c_{seq}(\beta), \beta) = B_0 + \beta \cdot b(c_{seq}(\beta)) = c_{seq}(\beta)$.[32]

Taking limits and using Assumption 3 gives us $\lim_{\beta\to 0} c_{seq} = B_0$ and $\lim_{\beta\to\infty} c_{seq} = 0$. It follows that $\lim_{\beta\to 0} \bar{x}_{seq} = \frac{\rho}{\bar{\pi}} + \left(\frac{2}{1+\bar{\pi}}\right)\left[\frac{B_0}{\Delta\pi}\right]$ and $\lim_{\beta\to\infty} \bar{x}_{seq} = \frac{\rho}{\bar{\pi}}$. This implies that $\lim_{\beta\to 0} \bar{x}_{seq}(\beta) > \bar{x}_{indv} > \bar{x}_{sim} > \lim_{\beta\to\infty} \bar{x}_{seq}(\beta)$. Differentiating $\bar{x}_{seq}(\beta)$ gives us $\frac{dx_{seq}}{d\beta} = \frac{2}{1+\bar{\pi}}\left[\frac{1}{\Delta\pi}\frac{dc_{seq}}{d\beta}\right] \leqslant 0$ given that $\frac{dc_{seq}}{d\beta} = \frac{b(c_{seq})}{1-\beta b'(c_{seq})} \leqslant 0$. $\hat{\beta}_1$ and $\hat{\beta}_2$ are defined by $\bar{x}_{seq}(\hat{\beta}_1) = \bar{x}_{indv}$ and $\bar{x}_{seq}(\hat{\beta}_2) = \bar{x}_{sim}$. It follows that for a sufficiently effective peer-influence function $\beta \in (\hat{\beta}_2, \infty)$, the lender would lend to projects $\bar{x} \in [\bar{x}_{indv}, \infty)$ under all three mechanisms, projects $\bar{x} \in [\bar{x}_{sim}, \bar{x}_{indv})$ under simultaneous and sequential lending and projects $\bar{x} \in [\bar{x}_{seq}, \bar{x}_{sim})$ only under sequential lending.[33]

[32] The analysis here is done for $\varepsilon = 0$ but could be done for any arbitrary $\varepsilon \in (0, 1]$.

[33] In section 3.1 we had assumed $\rho \geqslant \frac{\bar{\pi}\pi B_0}{(\Delta\pi)^2}$. For a given environment with $\tilde{\beta}$, if $c_{seq}(\tilde{\beta}) \geqslant$

There is no peer influence in individual and simultaneous lending and the borrowers' payoffs don't vary with β. In sequential lending the borrowers' expected payoff depend on c_{seq}, which is decreasing in β. Conversely, the expected output per unit of capital lent by the lender is lower in sequential lending as compared to individual and simultaneous lending. This is because the lender finds it optimal to stop lending if the first borrower fails. Thus, when $\beta \to 0$, the borrower's expected payoff in sequential lending is very high and $\bar{x}_{seq}(\beta) > \bar{x}_{indv} > \bar{x}_{sim}$. As β increases, $\bar{x}_{seq}$ decreases and we find that for a sufficiently high β, i.e., $\beta > \hat{\beta}_2$, $\bar{x}_{indv} > \bar{x}_{sim} > \bar{x}_{seq}(\beta)$. Further, with an extremely effective peer-influence function, unlike simultaneous and individual lending, sequential lending approaches the first best, i.e., $\lim_{\beta\to\infty} \bar{x}_{seq} = \frac{\rho}{\bar{\pi}}$.

Appendix

A Simultaneous Group Lending

We analyse the game described below for a given contract $(b_{ss}, b_{sf}, b_{fs}, b_{ff})$. For a subgame $\xi(c_1, c_2)$, B_1 and B_2's respective payoffs from exerting effort e_1 and e_2 respectively are $\Pi_1[e_1, e_2, c_1, c_2] = E[b_{ij} \mid e_1, e_2] - c_1 + \left[\frac{\bar{\pi}-\pi_1}{\bar{\pi}-\underline{\pi}}\right] B(c_2)$ and $\Pi_2[e_1, e_2, c_1, c_2] = E[b_{ij} \mid e_1, e_2] - c_2 + \left[\frac{\bar{\pi}-\pi_2}{\bar{\pi}-\underline{\pi}}\right] B(c_1)$ where for each borrower

$\frac{(1+\bar{\pi})B_0}{2}$, we need an additional assumption $\rho \geqslant \frac{2}{1+\bar{\pi}}\left[\frac{\bar{\pi}\underline{\pi}c_{seq}(\tilde{\beta})}{(\Delta\pi)^2}\right]$ to ensure that the high effort sequential lending contract yields a lower project productivity lower-bound than the individual lending low effort contract discussed in section 3.1. Given that $\frac{dc_{seq}}{d\beta} \leqslant 0$, this additional assumption is needed for $\beta \in [0, \breve{\beta})$ where $c_{seq}(\breve{\beta}) = \frac{(1+\bar{\pi})B_0}{2}$. For a $\beta \in [\breve{\beta}, \infty)$ the assumption $\rho \geqslant \frac{\bar{\pi}\underline{\pi}}{\Delta\pi}\left[\frac{B_0}{\Delta\pi}\right]$ suffices. (*See Referees' Appendix C.2*).

$k = \{1,2\}$, $\pi_k = \bar{\pi}$ if $e_k = H$ and $\pi_k = \underline{\pi}$ if $e_k = L$.[34]

We first analyse a subgame $\xi(c_1, c_2)$, where $c_1, c_2 \in [0, \infty)$, of the game described in Section 4.1 before moving up the tree. In the subgame $\xi(c_1, c_2)$, B_1 has no incentive to deviate from $HH(c_1, c_2)$ if $E[b_{ij} \,|\, HH] - E[b_{ij} \,|\, LH] \geqslant B(c_2)$ and B_2 has no incentive to deviate from $HH(c_1, c_2)$ if $E[b_{ij} \,|\, HH] - E[b_{ij} \,|\, LH] \geqslant B(c_1)$. Thus, it follows that $HH(c_1, c_2)$ is a Nash equilibrium if

$$E[b_{ij} \,|\, HH] - E[b_{ij} \,|\, LH] \geqslant \max\left[B(c_1), B(c_2)\right]. \tag{6}$$

Similarly, B_1 and B_2 has no incentive to deviate from $LL(c_1, c_2)$ if $E[b_{ij} \,|\, HL] - E[b_{ij} \,|\, LL] \leqslant B(c_2)$ and $E[b_{ij} \,|\, HL] - E[b_{ij} \,|\, LL] \leqslant B(c_1)$. Thus, $LL(c_1, c_2)$ is a Nash equilibrium if

$$E[b_{ij} \,|\, HL] - E[b_{ij} \,|\, LL] \leqslant \min\left[B(c_1), B(c_2)\right]. \tag{7}$$

$HH(c_1, c_2)$ and $LL(c_1, c_2)$ are both Nash equilibria in subgame $\xi(c_1, c_2)$ if

$$\begin{aligned} E[b_{ij} \,|\, HL] - E[b_{ij} \,|\, LL] &\leqslant \min\left[B(c_1), B(c_2))\right] \\ &\leqslant \max\left[B(c_1), B(c_2))\right] \leqslant E[b_{ij} \,|\, HH] - E[b_{ij} \,|\, LH]. \end{aligned} \tag{8}$$

In this case, a borrower B_k with peer $B_{k'}$ would prefer $HH(c_1, c_2)$ over

[34] For ease of exposition, we use $\bar{e}_1\bar{e}_2(\bar{c}_1, \bar{c}_2)$ as a shorthand notation to refer to a particular outcome where B_1 and B_1 choose effort levels $e_1 = \bar{e}_1$ and $e_2 = \bar{e}_2$ respectively in the subgame $\xi(\bar{c}_1, \bar{c}_2)$. Thus, for instance, $LH(\bar{c}_1, \bar{c}_2)$ refers to a situation where B_1 and B_2 choose $c_1 = \bar{c}_1$ and $c_2 = \bar{c}_2$ at $t = 1$ and choose $e_1 = L$ and $e_2 = H$ at $t = 2$ respectively. Since we have assumed that the the project returns of borrowers in a group are statistically independent, the likelihood of state ss occurring with $e_1 = L$ and $e_2 = H$ is given by $\underline{\pi}\bar{\pi}$.

$LL(c_1, c_2)$ if $E[b_{ij} \,|\, HH] - c_k \geqslant E[b_{ij} \,|\, LL] - c_k + B(c_{k'})$. Both B_1 and B_2 would prefer $HH(c_1, c_2)$ over $LL(c_1, c_2)$ if

$$E[b_{ij} \,|\, HH] - E[b_{ij} \,|\, LL] \geqslant \max\big[B(c_1), B(c_2)\big]. \tag{9}$$

Lets roll back the game and analyse B_1 and B_2's simultaneous decision on c_1 and c_2 at $t = 1$. There are three possible cases, $c_1 < c_2$, $c_1 = c_2$ and $c_1 > c_2$. Let's start with the case where $c_1 < c_2$. This implies that $B(c_1) > B(c_2)$ and $\max[B(c_1), B(c_2)] = B(c_1)$ and $\min[B(c_1), B(c_2)] = B(c_2)$. From (6), $HH(c_1, c_2)$ is Nash equilibrium if $E[b_{ij} \,|\, HH] - E[b_{ij} \,|\, LH] \geqslant B(c_1)$. From (7), $LL(c_1, c_2)$ is the Nash equilibrium if $E[b_{ij} \,|\, HL] - E[b_{ij} \,|\, LL] \leqslant B(c_2)$. We know from (8) that if both $HH(c_1, c_2)$ and $LL(c_1, c_2)$ are Nash equilibria in subgame $\xi(c_1, c_2)$, then both borrowers will prefer $HH(c_1, c_2)$ over $LL(c_1, c_2)$ if $E[b_{ij} \,|\, HH] - E[b_{ij} \,|\, LL] \geqslant B(c_1)$. Given that $c_1 \in [0, \infty)$, from (6) and (9) we know that $HH(c_1, c_2)$ will always be the preferred Nash equilibrium in this subgame if condition (1), i.e., $E[b_{ij} \,|\, HH] - E[b_{ij} \,|\, LL] \geqslant B_0$ and condition (2), i.e., $E[b_{ij} \,|\, HH] - E[b_{ij} \,|\, LH] \geqslant B_0$ hold. It follows that for cases $c_1 > c_2$ and $c_1 = c_2$ where $c_1, c_2 \in [0, \infty)$, $HH(c_1, c_2)$ will always be the preferred Nash equilibrium if (1) and (2) is satisfied.

A.1 Optimal Contract in Simultaneous Group Lending

The lender effectively minimises $E[b_{ij} \,|\, HH]$ subject to constraints (1) and (2). The lender's problem (P_{sim}) can be written as the following Lagrangian.

$$
\begin{aligned}
\mathcal{L} = &-\left[\bar{\pi}^2 b_{ss} + \bar{\pi}(1-\bar{\pi})b_{sf} + \bar{\pi}(1-\bar{\pi})b_{fs} + (1-\bar{\pi})^2 b_{ff}\right] \\
&+\lambda\left[[\bar{\pi}+\underline{\pi}]\,b_{ss} + [1-(\bar{\pi}+\underline{\pi})]\,b_{sf} + [1-(\bar{\pi}+\underline{\pi})]\,b_{fs} - [2-(\bar{\pi}+\underline{\pi})]\,b_{ff} - \frac{B_0}{\Delta\pi}\right] \\
&+\mu\left[\bar{\pi}b_{ss} + (1-\bar{\pi})\,b_{sf} - \bar{\pi}b_{fs} - (1-\bar{\pi})b_{ff} - \frac{B_0}{\Delta\pi}\right]
\end{aligned}
$$

The first order conditions are given below.

$$\frac{\partial\mathcal{L}}{\partial b_{ss}} = -\bar{\pi}^2 + \lambda(\bar{\pi}+\underline{\pi}) + \mu\bar{\pi} \tag{10}$$

$$\frac{\partial\mathcal{L}}{\partial b_{sf}} = -\bar{\pi}(1-\bar{\pi}) + [1-(\bar{\pi}+\underline{\pi})]\,\lambda + (1-\bar{\pi})\mu \tag{11}$$

$$\frac{\partial\mathcal{L}}{\partial b_{fs}} = -\bar{\pi}(1-\bar{\pi}) + [1-(\bar{\pi}+\underline{\pi})]\,\lambda - \bar{\pi}\mu \tag{12}$$

$$\frac{\partial\mathcal{L}}{\partial b_{ff}} = -(1-\bar{\pi})^2 - [2-(\bar{\pi}+\underline{\pi})]\lambda - (1-\bar{\pi})\mu \tag{13}$$

(10) and (12) give us $\lambda = \bar{\pi}$ and $\mu = -\underline{\pi}$. Evaluating the first order conditions at $\lambda^* = \bar{\pi}$ and $\mu^* = 0$, we find that $\frac{\partial\mathcal{L}}{\partial b_{ss}} = \bar{\pi}\underline{\pi} > 0$, $\frac{\partial\mathcal{L}}{\partial b_{sf}} = -\bar{\pi}\underline{\pi} < 0$, $\frac{\partial\mathcal{L}}{\partial b_{fs}} = -\bar{\pi}\underline{\pi} < 0$ and $\frac{\partial\mathcal{L}}{\partial b_{f}} = -(1-\bar{\pi})^2 - [2-(\bar{\pi}+\underline{\pi})]\bar{\pi} < 0$. Thus, (1) binds and (2) is slack and it is optimal to set $b_{sf} = b_{fs} = b_{ff} = 0$ and $b_{ss} = \frac{B_0}{\bar{\pi}^2-\underline{\pi}^2} > 0$. $c_1 = c_2 = 0$ follows from the proof above.

B Sequential Group Lending

At $t = 3$, B_1's project outcome is realised. Subgames $\xi(c_2, e_1, s, \ldots)$ are where B_1's project succeeds and subgames $\xi(c_2, e_1, f, \ldots)$ are where B_1's project 372
fails.[35] (c_1^s, e_2^s) and (c_1^f, e_2^f) are defined as the peer-influence intensity B_1 and effort B_2 chooses in subgames $\xi(c_2, e_1, s)$ and $\xi(c_2, e_1, f)$ respectively.[36]

Let's first analyse the case where B_1's project has succeeded. In subgame $\xi(c_2, e_1, s, c_1^s)$, B_2 chooses $e_2^s = H$ at $t = 5$ if $\bar{\pi} b_{ss} + (1 - \bar{\pi}) b_{sf} \geqslant \underline{\pi} b_{ss} + (1 - \underline{\pi}) b_{sf} + B(c_1)$. This condition holds if

$$b_{ss} - b_{sf} - \frac{B(c_1^s)}{\Delta\pi} \geqslant 0. \tag{14}$$

In subgame $\xi(c_2, e_1, s)$, B_1 chooses c_1^s at $t = 4$. Let's define $c_1^{s\prime} = B^{-1}\left[\Delta\pi(b_{ss} - b_{sf})\right]$ from (14). In making her decision, B_1 faces the payoff function $[\bar{\pi} b_{ss} + (1 - \bar{\pi}) b_{sf} - c_1^s]$ if $c_1^s \in \left[c_1^{s\prime},\ \infty\right)$ and payoff function $[\underline{\pi} b_{ss} + (1 - \underline{\pi}) b_{sf} - c_1^s]$ if $c_1^s \in \left[0,\ c_1^{s\prime}\right)$. B_1's payoff function above at $t = 4$ is non-monotonic in c_1^s and discontinuous at $c_1^{s\prime}$. This implies that B_1 effectively faces a binary choice where by choosing $c_1^s = 0$ would lead B_2 to choose $e_2 = L$ and choosing $c_1^s = c_1^{s\prime}$ would lead to B_2 choosing $e_2 = H$. B_1 would choose $c_1^s = c_1^{s\prime}$ if the following condition holds.

$$b_{ss} - b_{sf} - \frac{c_1^{s\prime}}{\Delta\pi} \geqslant 0 \tag{15}$$

[35] Nature chooses either s or f for B_1's project.

[36] In subgame $\xi(c_2, e_1, s)$, both borrowers get payoff b_{ss} if B_2's project succeeds and b_{sf} if it fails. In subgame $\xi(c_2, e_1, f)$, both borrowers get payoff b_{fs} if B_2's project succeeds and b_{ff} if it fails.

(14) and (15) can be summarised as condition (3). If (3) is satisfied, then in the subgame $\xi(c_2, e_1, s, \ldots)$ B_1 will always choose $c_1^s \geqslant c_1^{s\prime} = B^{-1}[\Delta\pi(b_{ss} - b_{sf})]$, which will ensure that B_2 has the incentive to choose $e_2^s = H$. Let's now analyse the case where B_1's project has failed. In subgame $\xi(c_2, e_1, f, c_1)$, B_2 chooses $e_2^f = H$ at $t = 5$ if $\bar{\pi}b_{fs} + (1-\bar{\pi})b_{ff} \geqslant \underline{\pi}b_{fs} + (1-\underline{\pi})b_{ff} + B(c_1)$. This condition holds if

$$b_{fs} - b_{ff} - \frac{B(c_1^f)}{\Delta\pi} \geqslant 0. \tag{16}$$

In subgame $\xi(c_2, e_1, f)$, B_1 chooses c_1^f at $t = 4$. Let's define $c_1^{f\prime\prime} = B^{-1}\left[\Delta\pi(b_{fs} - b_{ff})\right]$ from (16). In making her decision, B_1 faces the payoff function $\left[\bar{\pi}b_{fs} + (1-\bar{\pi})b_{ff} - c_1^f\right]$ if $c_1^f \in \left[c_1^{f\prime\prime}, \infty\right)$ and payoff function $\left[\underline{\pi}b_{fs} + (1-\underline{\pi})b_{ff} - c_1^f\right]$ if $c_1^f \in \left[0, c_1^{f\prime\prime}\right)$. B_1 would choose $c_1^f = c_1^{f\prime\prime}$ if the following condition holds.

$$b_{ss} - b_{sf} - \frac{c_1^{f\prime\prime}}{\Delta\pi} \geqslant 0 \tag{17}$$

(16) and (17) can be summarised as condition (4).

In analysing subgame $\xi(c_2)$, we have to incorporate the expectations that the games continues with probability ε if B_1's project fails. In subgame $\xi(c_2)$, B_1 chooses $e_1 = H$ at $t = 2$ if $\bar{\pi}[\bar{\pi}b_{ss} + (1-\bar{\pi})b_{sf}] + \varepsilon(1-\bar{\pi})[\bar{\pi}b_{fs} + (1-\bar{\pi})b_{ff}] + (1-\varepsilon)(1-\bar{\pi})b_f - c_1 \geqslant \underline{\pi}[\bar{\pi}b_{ss} + (1-\bar{\pi})b_{sf}] + \varepsilon(1-\underline{\pi})[\bar{\pi}b_{fs} + (1-\bar{\pi})b_{ff}] + (1-\varepsilon)(1-\bar{\pi})b_f - c_1 + B(c_2)$. This condition holds if

$$\bar{\pi}(b_{ss} - \varepsilon b_{fs}) + (1-\bar{\pi})(b_{sf} - \varepsilon b_{ff}) - (1-\varepsilon)b_f - \frac{B(c_2)}{\Delta\pi} \geqslant 0. \tag{18}$$

B_2 chooses c_2 at $t = 1$. Let's define as $c''' = B^{-1}\big[\Delta\pi[\bar{\pi}(b_{ss} - \varepsilon b_{fs}) + (1 - \bar{\pi})(b_{sf} - \varepsilon b_{ff}) - (1 - \varepsilon)b_f]\big]$ from (18). B_2 effectively faces a binary choice where by choosing $c_2 = 0$ would lead B_1 to choose $e_1 = L$ and choosing $c_2 = c'''$ would lead to B_1 choosing $e_1 = H$. B_2 would choose $c_2 = c'''$ if the following condition holds.

$$\bar{\pi}(b_{ss} - \varepsilon b_{fs}) + (1 - \bar{\pi})(b_{sf} - \varepsilon b_{ff}) - (1 - \varepsilon)b_f - \frac{c_2}{\Delta\pi} \geqslant 0 \qquad (19)$$

(18) and (19) can be summarised as condition (5).

B.1 Optimal Contract in Sequential Group Lending

The lender minimises $E[b_{ij} \mid HH]$ subject to (3), (4) and (5). The lender's problem ($\mathrm{P_{seq}}$) can be written as the following Lagrangian.

$$\begin{aligned}
\mathcal{L} = &- \left[\bar{\pi}^2 b_{ss} + \bar{\pi}(1 - \bar{\pi})(b_{sf} + \varepsilon b_{fs}) + (1 - \bar{\pi})^2 \varepsilon b_{ff} + (1 - \varepsilon)(1 - \bar{\pi})b_f\right] \\
&+ \lambda_1 \left[(b_{ss} - b_{sf}) - \frac{c_1^s}{\Delta\pi}\right] + \mu_1 \left[(b_{ss} - b_{sf}) - \frac{B(c_1^s)}{\Delta\pi}\right] \\
&+ \lambda_2 \left[(b_{fs} - b_{ff}) - \frac{c_1^f}{\Delta\pi}\right] + \mu_2 \left[(b_{fs} - b_{ff}) - \frac{B(c_1^f)}{\Delta\pi}\right] \\
&+ \lambda_3 \left[\bar{\pi}(b_{ss} - \varepsilon b_{fs}) + (1 - \bar{\pi})(b_{sf} - \varepsilon b_{ff}) - (1 - \varepsilon)b_f - \frac{c_2}{\Delta\pi}\right] \\
&+ \mu_3 \left[\bar{\pi}(b_{ss} - \varepsilon b_{fs}) + (1 - \bar{\pi})(b_{sf} - \varepsilon b_{ff}) - (1 - \varepsilon)b_f - \frac{B(c_2)}{\Delta\pi}\right]
\end{aligned}$$

First order conditions are given below.

$$\frac{\partial \mathcal{L}}{\partial c_1^s} = -\left[\frac{\lambda_1 + \mu_1 B'(c_1^s)}{\Delta\pi}\right] \tag{20}$$

$$\frac{\partial \mathcal{L}}{\partial c_1^f} = -\left[\frac{\lambda_2 + \mu_2 B'(c_1^f)}{\Delta\pi}\right] \tag{21}$$

$$\frac{\partial \mathcal{L}}{\partial c_2} = -\left[\frac{\lambda_3 + \mu_3 B'(c_2)}{\Delta\pi}\right] \tag{22}$$

$$\frac{\partial \mathcal{L}}{\partial b_{ss}} = -\bar{\pi}^2 + (\lambda_1 + \mu_1 + \lambda_2 + \mu_2) + \bar{\pi}(\lambda_3 + \mu_3) \tag{23}$$

$$\frac{\partial \mathcal{L}}{\partial b_{sf}} = -\bar{\pi}(1-\bar{\pi}) - (\lambda_1 + \mu_1) + (1-\bar{\pi})(\lambda_3 + \mu_3) \tag{24}$$

$$\frac{\partial \mathcal{L}}{\partial b_{fs}} = -\varepsilon\bar{\pi}(1-\bar{\pi}) + (\lambda_2 + \mu_2) - \varepsilon(\lambda_3 + \mu_3) \tag{25}$$

$$\frac{\partial \mathcal{L}}{\partial b_{ff}} = -\varepsilon(1-\bar{\pi})^2 - (\lambda_2 + \mu_2) - \varepsilon(1-\bar{\pi})(\lambda_3 + \mu_3) \tag{26}$$

$$\frac{\partial \mathcal{L}}{\partial b_f} = -(1-\varepsilon)(1-\bar{\pi}) - (1-\varepsilon)(\lambda_3 + \mu_3) \tag{27}$$

The first order conditions can be solved to gives us $\lambda_1^* = \mu_1^* = 0$,[37] $\mu_2^* = \left[\frac{\bar{\pi}(2-\bar{\pi})\varepsilon}{1+\varepsilon}\right]\left[\frac{1}{1-B'(c_1^f)}\right]$, $\lambda_2^* = \left[\frac{\bar{\pi}(2-\bar{\pi})\varepsilon}{1+\varepsilon}\right]\left[\frac{B'(c_1^f)}{1-B'(c_1^f)}\right]$ $\mu_3^* = \left[\frac{\bar{\pi}[1-\varepsilon(1-\bar{\pi})]}{1+\varepsilon}\right]\left[\frac{1}{1-B'(c_2)}\right]$ and $\lambda_3^* = \left[\frac{\bar{\pi}[1-\varepsilon(1-\bar{\pi})]}{1+\varepsilon}\right]\left[\frac{-B'(c_2)}{1-B'(c_2)}\right]$. Using these values, we can show from (26) and (27) that $\frac{\partial \mathcal{L}}{\partial b_f} < 0$ and $\frac{\partial \mathcal{L}}{\partial b_{ff}} < 0$ given that $\lambda_2^*, \mu_2^*, \lambda_3^*, \mu_3^* > 0$. This implies that the optimal value of $b_f = b_{ff} = 0$.

$\lambda_2^*, \mu_2^* > 0$ imply that both components of the constraint (4) bind. Let's

[37] $\mu_1 = \left[\frac{-\bar{\pi}(1-\bar{\pi})(2-\bar{\pi})\varepsilon}{1+\varepsilon}\right]\left[\frac{1}{1-B'(c_1^s)}\right]$ $\lambda_1 = \left[\frac{-\bar{\pi}(1-\bar{\pi})(2-\bar{\pi})\varepsilon}{1+\varepsilon}\right]\left[\frac{B'(c_1^s)}{1-B'(c_1^s)}\right]$.

define $c_{seq} = B(c_{seq})$. Both components of (4) binding gives us

$$b_{fs} = \frac{c_{seq}}{\Delta\pi}. \tag{28}$$

$\lambda_3^*, \mu_3^* > 0$ imply that both components of the constraint (5) bind. This along with (28) give us

$$\bar{\pi} b_{ss} + (1 - \bar{\pi}) b_{sf} = [1 + \bar{\pi}\varepsilon] \frac{c_{seq}}{\Delta\pi}. \tag{29}$$

$\lambda_1^* = \mu_1^* = 0$ imply that (3) remains slack and will be satisfied if $b_{ss} - b_{sf} \geqslant \frac{c_{seq}}{\Delta\pi}$. Any contract $(b_{ss}, b_{sf}, b_{fs}, 0, 0)$ that satisfies (3), (28) and (29) solves the problem ($\mathrm{P_{seq}}$).

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C Appendix for Referees

C.1 Sequential Group Lending: Break Even Condition

This elaborates on the discussion on page 15 following Proposition 3 in section 5.

Expected output: $\bar{\pi}\Big[(1+\bar{\pi})+(1-\bar{\pi})\varepsilon\Big]\bar{x}$

$2\bar{x}$ with probability $\bar{\pi}^2$ (both succeeds)

$\bar{x}$ with probability $\bar{\pi}(1-\bar{\pi})$ (B_1 succeeds and B_2 fails)

$\bar{x}$ with probability $(1-\bar{\pi})\bar{\pi}\varepsilon$ (B_1 fails but the game continues)

Expected capital use: $\Big[(1+\bar{\pi})+(1-\bar{\pi})\varepsilon\Big]\rho$

B_1 succeeds: 2ρ with probability $\bar{\pi}$

B_1 fails:

and game continues ... 2ρ with probability $(1-\bar{\pi})\varepsilon$

and the game terminates ... ρ with probability $(1-\bar{\pi})(1-\varepsilon)$

$$\begin{aligned}&\Big[\bar{\pi}(2\rho)+(1-\bar{\pi})\varepsilon(2\rho)+(1-\bar{\pi})(1-\varepsilon)\rho\Big]\\ =\;&\Big[(1+\bar{\pi})+(1-\bar{\pi})\varepsilon\Big]\rho\end{aligned}$$

Expected Rents ($b_{ff}=b_f=0$):

$$\begin{aligned}&= 2\left[\bar{\pi}^2 b_{ss}+\bar{\pi}(1-\bar{\pi})(b_{sf}+\varepsilon b_{fs})\right]\\ &= 2\Big[\bar{\pi}\big[\bar{\pi}b_{ss}+(1-\bar{\pi})b_{sf}\big]+\bar{\pi}(1-\bar{\pi})\varepsilon b_{fs}\Big]\\ &= 2\left[\bar{\pi}(1+\bar{\pi}\varepsilon)+\bar{\pi}(1-\bar{\pi})\varepsilon\right]\left[\frac{c_{seq}}{\Delta\pi}\right]\\ &= 2\bar{\pi}\big[1+\varepsilon\big]\frac{c_{seq}}{\Delta\pi}\end{aligned}$$

Break-even condition

$$\bar{\pi}\Big[(1+\bar{\pi})+(1-\bar{\pi})\varepsilon\Big]\bar{x} \geqslant \Big[(1+\bar{\pi})+(1-\bar{\pi})\varepsilon\Big]\rho + 2\bar{\pi}\big[1+\varepsilon\big]\frac{c_{seq}}{\Delta\pi}$$

$$\bar{x} \geqslant \frac{\rho}{\bar{\pi}} + \frac{2[1+\varepsilon]}{(1+\bar{\pi})+(1-\bar{\pi})\varepsilon}\left[\frac{c_{seq}}{\Delta\pi}\right]$$

Differentiating $\bar{x}$ with respect with ε

$$\frac{d\bar{x}_{seq}}{d\varepsilon} = \frac{\big[(1+\bar{\pi})+(1-\bar{\pi})\varepsilon\big]2 - 2[1+\varepsilon](1-\bar{\pi})}{\big[(1+\bar{\pi})+(1-\bar{\pi})\varepsilon\big]^2}\left[\frac{c_{seq}}{\Delta\pi}\right]$$

$$= \frac{2[2\bar{\pi}]}{\big[(1+\bar{\pi})+(1-\bar{\pi})\varepsilon\big]^2}\left[\frac{c_{seq}}{\Delta\pi}\right] \geqslant 0$$

C.2 Interest Rate Lower Bound

High effort would allow lender to lend to lower the *productivity lower-bound* if the following condition holds in the various lending mechanisms. 456

Individual Lending:

$$\frac{\rho}{\underline{\pi}} \geqslant \frac{\rho}{\bar{\pi}} + \frac{B(0)}{\Delta\pi}$$

$$\rho \geqslant \left[\frac{\bar{\pi}\underline{\pi}B(0)}{(\Delta\pi)^2}\right] = \underline{\rho}_{indv}$$

Simultaneous Group Lending:

$$\frac{\rho}{\underline{\pi}} \geqslant \frac{\rho}{\bar{\pi}} + \left(\frac{\bar{\pi}}{\bar{\pi}+\underline{\pi}}\right)\left[\frac{B(0)}{\Delta\pi}\right]$$

$$\rho \geqslant \left(\frac{\bar{\pi}}{\bar{\pi}+\underline{\pi}}\right)\left[\frac{\bar{\pi}\underline{\pi}B(0)}{(\Delta\pi)^2}\right] = \underline{\rho}_{sim}$$

Sequential Group Lending:

$$\frac{\rho}{\underline{\pi}} \geqslant \frac{\rho}{\bar{\pi}} + \frac{2}{1+\bar{\pi}} \left[\frac{c_{seq}(\beta)}{\Delta\pi} \right]$$
$$\rho \geqslant \frac{2}{1+\bar{\pi}} \left[\frac{\bar{\pi}\underline{\pi} c_{seq}(\beta)}{(\Delta\pi)^2} \right] = \underline{\rho}_{seq}(\beta)$$

As $\beta \to 0$, $\rho \geqslant \frac{2}{1+\bar{\pi}} \left[\frac{\bar{\pi}\underline{\pi} B(0)}{(\Delta\pi)^2} \right]$ and as $\beta \to \infty$, $\rho \geqslant 0$.

This establishes that there is a lower bound of ρ for which high effort contract in each mechanism yields the smallest *productivity lower-bound*. It is clear that $\underline{\rho}_{indv} \geqslant \underline{\rho}_{sim}$. $\underline{\rho}_{seq}(\beta)$ maybe greater than $\underline{\rho}_{indv}$ for $\beta \in [0, \breve{\beta})$ where $c_{seq}(\breve{\beta}) = \frac{(1+\bar{\pi})B_0}{2}$. This is further explored in Footnote 33 in section 5.1 on page 17.