Sequential Group Lending with Moral Hazard

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Abstract

The paper examines the pros and cons of lending sequentially to a group, composed of two wealthless individuals who are jointly liable for each other’s project outcome. Sequential lending entails lending to one borrower per period, with the proviso that the second borrower’s loan is contingent on the first borrower’s project succeeding. We show that in a moral hazard environment, where the borrowers can influence their peer’s effort through costly monitoring, the borrowers are allocated smaller rents in sequential as compared to simultaneous group lending. Conversely, the lender’s capital is less productive in sequential as compared to simultaneous lending. Thus, for a sufficiently efficient peer monitoring technology a greater range of projects is feasible under sequential lending.

Keywords: Group lending, joint liability, peer monitoring, sequential finance, microfinance

JEL Classification: D82, G20, O12, O2

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1 Introduction

The paper examines whether the lender can use the timings of the loans to increase the efficiency of peer monitoring when lending to a group of jointly liable impoverished individuals. We show that disbursing the loans in a sequence to a group can help the lender save on the economic rents left to the collateral-less borrowers. The saving on economic rents can potentially allow the lender to finance a greater range of projects.

Specifically, we compare sequential and simultaneous group lending mechanisms in a moral hazard environment where there are two costly tasks, namely, effort on a project and monitoring the peer. We assume that a borrower is able to influence her peer’s effort decision by monitoring her. A group is composed of two aspiring collateral-less borrowers. In simultaneous group lending, both borrowers in the group receive their loans simultaneously from the lender. Conversely, in sequential group lending, the lender disburses the loans sequentially within the group with the proviso that the second borrower obtains the loan only if the first borrower’s project succeeds.

In simultaneous group lending, the borrowers make their decisions on their respective tasks\(^1\) simultaneously. Consequently, along with incentivizing the tasks individually\(^2\), the lender also has to incentivize the group’s actions collectively. This is done by satisfying the group’s collective incentive compatibility condition.\(^3\) In the moral hazard literature on group lending, this has been a recurrent theme hitherto in papers like Conning (1996), Conning

\(^1\)The tasks of undertaking effort for her own project and monitoring her peer.
\(^2\)When a task is incentivized individually, each individual borrower in the group is given the requisite incentive to undertake that specific task, given that her peer is undertaking her tasks satisfactorily (from the lender’s perspective).
\(^3\)The group’s collective incentive compatibility condition can be thought of as the incentive compatibility condition for the group when it is operating as a single cogent entity.
We use this as a point of departure to analyse sequential group lending. We show that lending sequentially allows the lender to temporally separate the borrowers’ decisions on their respective tasks. As a result, the lender only incentivises the borrowers’ tasks individually and not collectively, leaving the group’s collective incentive compatibility condition slack. Consequently, the advantage of lending sequentially is that the lender is able to lower the rents allocated to the borrowers.

The disadvantage of lending sequentially is that punishing the whole group, if the first borrower’s project fails, lowers the productivity of the lender’s capital. We show that for a sufficiently efficient peer monitoring technology, the lender is able to finance a greater range of projects with sequential as compared to simultaneous group lending.

Further, this framework allows us to examine how the borrowers’ ability to side contract on actions amongst themselves affects group lending. Even without any explicit ability to side contract on actions, in simultaneous group lending, the group members obtain rents that they would have obtained if they had an ability to collude perfectly. This is because simultaneous group lending is only feasible if the group’s collective incentive compatibility condition is satisfied.³ (Proposition 2) Conversely, in sequential group lending, the group’s collective incentive compatibility condition remains slack. (Corollary

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³Monitoring, by itself, is an unobservable task. When the groups are encouraged to collude, they internalise the cost of monitoring. (Stiglitz (1990), Varian (1990) and Ghatak and Guinnane (1999)) If the lender instead sets out to explicitly encourage the borrowers to monitor, he can only do so through rents. Consequently, in a standard delegated monitoring model with wealth-less borrower and monitor, where the monitor can influence the borrower’s effort choice by monitoring, the lender incentivises effort and monitoring by allocating rents to the borrower and the monitor. Conning (1996) and Conning (2000) show that in (simultaneous) group lending one of these rents turn out to be the collusion rents. Further, they are able to show only the more expensive of the two rents have to be paid, leading to gains in lending efficiency.

⁵A result also obtained by Conning (1996) and Conning (2000).
1) In sequential lending, if the borrowers have an unlimited ability to side-contract on actions, the borrowers will be able to obtain the rents that they would have obtained in simultaneous group lending. Thus, lending sequentially to the group allows the lender to exploit the borrowers’ inability to side-contract on actions across time to lower the rents allocated to them.

In practice, the Grameen Bank (Bangladesh) follows the sequential group lending mechanism where borrowers receive their credit sequentially. Banco Solidario (Bolivia) and ACCION affiliated microfinance organisations allocate credit within the groups simultaneously.

In a theoretical paper, Varian (1990) has explored the benefits of sequential lending in a setup with heterogeneous borrowers, i.e. ones with high and low productivity. The critical assumption in the paper is that, given requisite incentives, the high productivity borrower can school the low productivity borrower and turn her into a high productivity borrower.

The paper shows that when lending to a group of randomly selected borrowers, the lender prefers to lend sequentially, as it increases his profits. He offers the second period borrower a contract only after observing the output of the first period borrower. If the first period borrower is the low type, schooling her helps the second period borrower get a favourable contract.

Lending sequentially increases the lender’s profit in two ways. First, the first period production signal helps him in sorting out the borrower’s type more effectively. Second, the information transmission increases the number of high productivity borrowers. The result of the paper, of course, rests on the assumption of perfect information transmission within groups.

Roy Chowdhury (2006) extends the Varian (1990) idea to analyse en-

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6See Armendáriz de Aghion and Morduch (2005, Pages 87-88) for a detailed description of Grameen Bank’s sequential group lending mechanism.

7of using the first period production signal in sequential lending to ascertain information
dogenous group formation in an environment with heterogeneous borrowers, i.e., some borrowers are vulnerable to peer sanctions while others are not. In an infinite horizon framework, the papers show that borrowers vulnerable to peer sanction prefer to group with their own type, if they care sufficiently about the future. Sequential lending thus allows the lender to screen the groups more efficiently using the signal from the first borrower’s outcome.

In a significant recent contribution, Roy Chowdhury (2005) finds that in a costly monitoring setup, group lending with joint liability does not necessarily alleviate the moral hazard problem. The intensity with which the group members choose to monitor their peer in the group are strategic complements i.e. monitoring by one borrower encourages the other borrower to monitor and vice versa. Roy Chowdhury (2005) finds that this strategic complementarity may lead to both borrowers choosing not to monitor their peer when they obtain credit simultaneously. In this case, lending sequentially enhances the incentives for peer monitoring and results in positive levels of monitoring.

The zero-monitoring result in simultaneous group lending in Roy Chowdhury (2005) is driven by the assumption that the interest rate (and the borrowers’ payoffs) on the borrowers’ loans is exogenously determined. Consequently, if the borrowers are allocated less than sufficient rents, simultaneous group lending becomes infeasible. (Roy Chowdhury, 2005, Proposition 2)

Simultaneous group lending has been shown to be viable in practice in Banco Solidario (Bolivia) and ACCION affiliated microfinance organizations amongst others. As discussed above, simultaneous group lending is feasible if the group can be encouraged to cooperate. This is achieved by satisfy-
ing the group’s collective incentive compatibility condition by allocating the borrowers sufficiently large rents. Thus, simultaneous group lending is feasible if the lender is free to vary the interest rate charged (and determine the borrowers’ payoffs). If the borrowers can influence their peer’s effort decision through monitoring, the lender will induce positive levels of monitoring in simultaneous group lending by allocating appropriate rents to the borrowers. This allows us to compare the cost (in terms of rents) of implementing the two lending mechanisms, i.e., sequential and simultaneous group lending.

In this paper, we confine ourselves to the problem of the borrower’s effort choice before the project is undertaken. Other papers in the microfinance literature have shown that joint liability group lending can alleviate information problems like adverse selection (Armendáriz de Aghion and Gollier (2000), Ghatak (2000), Laffont and NGuessan (2000) and Van Tassel (1999)) and strategic default (Besley and Coate (1995) and Che (1999)) associated with lending to the poor. Ghatak and Guinnane (1999) and Morduch (1999) are two excellent recent surveys in this area.

2 Model

There are two agents $B_1$ and $B_2$. Each of them has access to a project requiring a lump-sum investment of 1 unit of capital. The project produces an uncertain and observable outcome $x$, valued at $\bar{x}$ when it succeeds ($s$) and 0 when it fails ($f$).

2.1 Agents

The agents are risk neutral, with zero reservation wage and no wealth. Agents may choose to pursue the aforementioned project with a high ($H$) or low ($L$)
effort \(e\), which is unobservable. With a high (low) effort, \(\bar{x}\) is realised with a probability \(\pi^h (\pi^l)\) and 0 with \(1 - \pi^h (1 - \pi^l)\). \((\pi^h > \pi^l)\)

By exerting low effort, agents obtain private benefits of value \(B\) from the project which are non-pecuniary and non-transferable amongst the agents. The private benefits can be curtailed by monitoring, which is undertaken at a non-pecuniary cost \(c\) to the monitor.

The agents are able to monitor and curtail each other’s private benefits. The extent of monitoring is observable to the agents but unobservable to the lender. We impose the following assumption on the monitoring function \(B(c)\).

**Assumption 1** (Monitoring function \(B(c)\)).

i. \(B(0) > 0\)

ii. \(B(c)\) is continuous and at least once differentiable \(\forall c \geq 0\)

iii. \(B_c(c) < 0\) \(\forall c \geq 0\)

### 2.2 The Lender

The lender is risk-neutral. He is unable to monitor the agents and can only punish them through their payoffs. He can costlessly observe the initial capital invested in the project and the output from the project. Further, we assume that the lender has the ability to enforce the contracts once the project outcome(s) is (are) realised. He has access to capital at \(\rho\), the opportunity cost of capital. He operates in a competitive market and is unable to earn any rents on the funds he lends, thus making zero profits.
2.3 The Agent’s Payoff

In individual lending, the borrower borrows 1 unit of capital once she accepts the contract offered by the lender. The lender may choose to delegate the task of monitoring to another agent. The lender makes the borrower’s and the monitor’s payoff, $b_i$ and $w_i$ respectively, contingent on $i = \{s, f\}$, the borrower’s project outcome.

In group lending, the lender finances the projects of the group members $B_1$ and $B_2$ once they accept the group contract offered by the lender. We assume that both borrowers want to undertake identical projects and the lender offers them symmetrical contracts. The lender makes the borrower’s payoff $b_{ij}$ contingent on $i$ and $j$, the outcomes of $B_1$ and $B_2$’s projects, respectively, in the group contract.

In a joint liability group contract a borrower’s payoff is affected by her peer’s project outcome. ($b_{is} \neq b_{if}$) The lender can punish a borrower for her peer’s project failure by ensuring that $b_{is} \geq b_{if}$ for $i = \{s, f\}$ with at least one strict inequality.

3 Individual Lending

The individual borrower undertakes a project by borrowing 1 unit of capital from the lender if she accepts the lender’s contract ($b_s, b_f$).

3.1 First-Best

The perfect information case, where the lender can observe the borrower’s effort level, is examined as a benchmark. The lender offers the borrower a
contract \((b_s, b_f)\) that solves the following problem:

\[
\max_{b_i} E[x | H] - E[b_i | H] \\
E[b_i | H] \geq 0 \\
b_i \geq 0; \quad i = s, f
\] (1)

The borrower’s participation constraint (1) binds and the limited liability constraint (2) binds in state \(f\). The lender offers the borrower a contract where \(b_s = b_f = 0\). If the borrower accepts the contract, she is able to undertake the project. Using the lender’s feasibility condition given below,

\[
E[x | H] \geq E[b_i | H] + \rho,
\] (3)

we find that \(x_s \geq \frac{\rho}{\pi x}\), that is, the lender can finance all the socially viable projects.

### 3.2 Second-Best

With incomplete information, the borrower’s effort is unobservable to the lender. The lender needs to give the borrower an incentive to exert high effort by rewarding her sufficiently if her project succeeds.

\[
E[b_i | H] \geq E[b_i | L] + B(0)
\] (4)

The incentive compatibility constraint above ensures that the borrower is not worse off if she exerts a high effort level. With no monitoring, the private benefits are at their maximal value, \(B(0)\).
### 3.2.1 The Optimal Contract without Delegated Monitoring

The lender offers the borrower a contract \((b_s, b_f)\) that maximizes the lender’s payoff \(E[x \mid H] - E[b_i \mid H]\) subject to the borrower’s participation constraint (1), limited liability constraint (2) and incentive compatibility constraint (4). The lender offers the borrower a contract \(\left(\frac{B(0)}{\Delta \pi}, 0\right)\) where \(\Delta \pi = \pi^h - \pi^l\). The contract ensures that the incentive compatibility constraint (4) binds and the limited liability constraint (2) binds only for state \(f\). The borrower is left with a positive expected rent leaving her participation constraint slack.

The lender is unable to punish the borrower when the project fails because of the limited liability constraint. The borrower gets the requisite incentive for high effort through higher payoffs when the project succeeds. This allows the borrower to retain a strictly positive limited liability rent. (Laffont and Martimort, 2002, page 119) Using the lender’s feasibility constraint (3) we get \(x^s \geq \frac{\rho}{\pi^s} + \frac{B(0)}{\Delta \pi}\), the set of feasible projects. The lender is unable to finance projects \(\bar{x} \in \left[\frac{\rho}{\pi^s}, \frac{\rho}{\pi^s} + \frac{B(0)}{\Delta \pi}\right]\).

### 3.2.2 Delegated Monitoring

Group lending is plagued with the possibility of collusion between the borrowers in the group. Understanding how collusion is prevented in the delegated monitoring model helps us better understand how it can be prevented in group lending.

Like the effort level, the lender cannot observe the task of monitoring. If the lender delegates the task of monitoring, he makes the monitor’s payoff contingent on the borrower’s project outcome. This gives the monitor the requisite incentive to influence the borrower’s effort choice by monitoring her and curtailing her private benefits \(B\). This particular lending mechanism is partially akin to joint liability in group lending, where the two borrowers’
project outcomes affect each other’s payoffs.

The borrower’s and monitor’s contracts work in conjunction with each other. The borrower’s contract aims to influence her effort choice directly through her payoff. The lender is also able to influence the borrower’s effort choice indirectly through the monitor’s contract.

The lender’s problem is set out in Appendix A. We find that the borrower’s and monitor’s incentive compatibility constraints bind in the optimal contract. Their respective participation constraints remain slack and their limited liability constraints bind only in state $f$. The lender induces monitoring of intensity $c_{dm}$ by offering the borrower a contract $\left( \frac{B(c_{dm})}{\Delta \pi}, 0 \right)$ and the monitor a contract $\left( \frac{c_{dm}}{\Delta \pi}, 0 \right)$, where $c_{dm} = B_c^{-1}(-1)$.

The lender delegates the task of monitoring only if $B_c(0) < -1$, that is, the benefit of curtailing the borrower’s private benefit initially is not overwhelmed by the payoff allocated to the monitor. The lender induces monitoring till the marginal benefit from additional monitoring is matched by its marginal cost.

### 3.2.3 Collusion

We examine whether the borrower and the monitor could benefit from colluding on actions, if they were able to fully side-contract amongst themselves. Collusion would entail the borrower exerting low effort and the monitor not monitoring.

**Proposition 1.** If the borrower’s private benefits are non-pecuniary and non-transferable, the borrower and the monitor would not collude, even if they were able to fully side contract amongst themselves.

Let us assume that agents can fully side-contract amongst themselves costlessly. Agents would choose the monitoring intensity and the effort level
together in order to maximise their collective payoffs. Thus, the no-collusion condition for the borrowers given below compares the expected surplus from not-colluding with the expected surplus from colluding.

\[
E[b_i \mid H] + E[w_i \mid H] - c \geq E[b_i \mid L] + E[w_i \mid L]
\]

Using the monitor’s and the borrower’s contracts from section 3.2.2, we find that the no-collusion condition is always satisfied given that \( \frac{B(c_{dm})}{\Delta \pi} \geq 0 \). By not monitoring, the monitor lowers her expected surplus by a greater amount than the amount she saves in cost of monitoring. Consequently, the borrower and the monitor do not benefit from colluding. Conversely, if the private benefits were transferable, the borrower and the monitor would prefer to collude. In this case, the no-collusion condition is given by

\[
E[b_i \mid H] + E[w_i \mid H] - c \geq E[b_i \mid L] + E[w_i \mid L] + B(0)
\]

Using the contracts from section 3.2.2, we find that the no-collusion condition is never satisfied given that \( B(c) < B(0) \).

4 Group Lending

Limited liability restricts the lender’s ability to use the payoffs to punish a borrower when her project fails. Conversely, joint-liability allows the lender the use of payoffs to punish a successful borrower if her peer’s project fails. Consequently, a lender can use a joint-liability group-contract to give each borrower an explicit incentive to influence her peer’s effort decision by mon-
itoring her and thus reducing the likelihood of the peer’s project failing.

A groups consists of two borrowers, $B_1$ and $B_2$, seeking loans from the lender that would enable them to undertake their respective projects. We compare a group lending mechanism where borrowers borrow simultaneously with the one where they borrow sequentially.

4.1 Simultaneous Group Lending

The lender offers the borrowers a joint liability group contract. If they accept the contract, the borrowers obtain loans for their respective projects simultaneously.

With costless monitoring, the lender has to leave each borrower a smaller rent in group lending as compared to individual lending. That is because the group’s collective incentive compatibility condition gets satisfied with lower rents as compared to an individual’s incentive compatibility condition in this case. (Armendáriz de Aghion and Morduch, 2005, page 97)

We show below that with costly monitoring, the lender has to leave sufficient rents to satisfy both

1. the individual borrower’s incentive compatibility condition associated with effort when her peer exerts high effort and both borrowers monitor each other

2. the group’s collective incentive compatibility condition.

With costly monitoring, satisfying (2) requires simultaneously incentivizing both tasks, namely effort and monitoring, for the group as a whole. It should also be noted that in the case of costless monitoring, (1) is always satisfied if (2) is satisfied.
The lender can distinguish between the four states of the world, once the outcome of the projects are realised. These states \( \{ij\} \) are:

- \( ss \) \( B_1 \) and \( B_2 \)'s projects succeed
- \( sf \) \( B_1 \)'s project succeeds and \( B_2 \)'s project fails
- \( fs \) \( B_1 \)'s project fails and \( B_2 \)'s project succeeds
- \( ff \) \( B_1 \) and \( B_2 \)'s projects fail

The game is played in two stages. The agents simultaneously choose their monitoring intensities and their effort choices in the first and second stage respectively. They choose a pair of monitoring intensities \((c_1, c_2)\) in the first stage where \( c_k \) is the monitoring intensity chosen by \( B_k \). A given pair of monitoring intensities \((c_1, c_2)\) then determines the payoff structure of the subgame \( \xi(c_1, c_2) \) in effort decisions, in the second stage.

Let \( b_{ij} \) denote the borrower’s pecuniary payoff in state \( ij \). The timing of the game is as follows:

- \( t=0 \) The lender offers \( B_1 \) and \( B_2 \) an identical contract \((b_{ss}, b_{sf}, b_{fs}, b_{ff})\).
  
  If they accept the contract, the game continues. Otherwise, it terminates.

- \( t=1 \) \( B_1 \) and \( B_2 \) choose their respective monitoring intensities \( c_1 \) and \( c_2 \) simultaneously.

- \( t=2 \) \( B_1 \) and \( B_2 \) choose their respective effort levels \( e_1 \) and \( e_2 \) simultaneously.

- \( t=3 \) \( B_1 \) and \( B_2 \)'s project outcome is realised.
  
  Both borrowers get payoffs \( b_{ij} \) depending on the realised state \( ij \).

The limited liability constraint ensures that the borrower’s payoffs in the contract are non-negative.

\[
b_{ij} \geq 0 \text{ for } i, j = \{s, f\}
\]
Symmetry requires that
\[ b_{sf} = b_{fs} \]  \hspace{1cm} (6)

**Assumption 2.** *The project returns are statistically independent.*

So, for instance, if \( B_1 \) exerts high effort and \( B_2 \) exerts low effort, the likelihood of state \( ss \) is \( \pi^l \pi^h \).

From the lender's perspective, the desired outcome of the game is one where both borrowers choose to exert high effort on their respective projects. \( ss \) and \( ff \) are the two most informative states for the lender. If \( ss \) occurs, the two agents are most likely to have undertaken the requisite monitoring to induce high effort from their respective peer. If \( ff \) occurs, the opposite is true. Consequently, the lender should reward \( ss \) and punish \( ff \) to the maximum extent possible.

The limited liability constraint (5) binds for \( ff \) leaving \( b_{ff} = 0 \). The lender can choose to allocate rewards to the remaining states. Increasing \( b_{ss} \) sharpens the incentive for the borrowers to make the desired outcome more likely. Given that the borrowers are risk neutral, it is optimal for the lender to reward only \( ss \) and leave no reward for any other states. Thus, the lender offers each agent a contract \((b_{ss}, 0, 0, 0)\).

We show in Appendix B that if the following two conditions are met, the lender’s desired outcome is the pure strategy subgame perfect nash equilibrium (SPNE) of the game.

The first condition is \( B_1 \)'s (and symmetrically \( B_2 \)'s) incentive compatibility constraint for effort level in the subgame \( \xi(c, c) \) where \( c \geq 0 \).\(^{10}\)

\[
E(b_{ij} \mid HH) - c \geq E(b_{ij} \mid LH) + B(c)
\]

\(^{10}\)\(E(b_{ij} \mid LH) = \pi^l \pi^h b_{ss}\) is the expected payoff borrower \( B_1 \) (and by symmetry \( B_2 \)) gets when \( B_1 \) (\( B_2 \)) exerts low effort and \( B_2 \) (\( B_1 \)) exerts high effort. See Assumption 2.
Once the borrowers have decided on their monitoring intensities $c$, and $B_2$ ($B_1$) has chosen to exert high effort, this condition ensures that $B_1$ ($B_2$) is no worse off exerting high effort as compared to exerting low effort. This condition is satisfied if

$$b_{ss} \geq \frac{B(c)}{\pi h \Delta \pi}$$  \hspace{1cm} (Condition 1)

Thus, monitoring makes inducing high effort cheaper for the lender. The second condition is the group’s collective incentive compatibility condition which ensures that the borrowers prefer the outcome where both monitor with intensity $c > 0$ and exert high effort over the outcome where both borrowers do not monitor and exert low effort.

$$E(b_{ij} | HH) - c \geq E(b_{ij} | LL) + B(0)$$  \hspace{1cm} (7)

That is, by undertaking requisite monitoring and exerting high effort, the agents are no worse off than they would have been if they had not monitored at all and exerted low effort. This condition is satisfied if

$$b_{ss} \geq \frac{B(0) + c}{\pi h \Delta \pi}$$  \hspace{1cm} (Condition 2)

It should be noted that the payoff required to satisfy condition 2 increases with $c$. Thus, the greater the monitoring intensity the lender wants to induce, the more expensive it is to satisfy the group’s collective incentive compatibility condition. We should also note that not allocating the borrowers sufficient rents to satisfy condition 2 is what made simultaneous group lending infeasible in Roy Chowdhury (2005, Proposition 2, page 423). With these two conditions satisfied, simultaneous group lending would always be feasible.
It is interesting to note that the payoffs that satisfy Condition 2 depend on \( B(0) \), the private benefits without monitoring and not on \( B(c) \), the private benefits after monitoring. Although, within a group, monitoring makes incentivizing the individual effort cheaper at the margin (Condition 1), it makes satisfying both tasks collectively more expensive (Condition 2). We summarise with the following proposition.

**Proposition 2.** Simultaneous group lending is feasible if the borrowers are allocated rents which satisfy conditions 1 and 2.

The lender’s problem follows:

\[
\max_{b_{ij}} E[x \mid H] - E[b_{ij} \mid H]
\]

subject to \( b_{ss} \geq \frac{1}{\pi^h \Delta \pi} \max \left[ B(c), \alpha (B(0) + c) \right] \) \hfill (8)

where \( \alpha = \frac{\pi^h}{\pi^h + \pi^l} \). To minimise the rents that the borrowers retain, the lender induces monitoring intensity \( c_{sim} \) defined by

\[
B(c_{sim}) = \alpha (B(0) + c_{sim})
\] \hfill (9)

The borrower’s expected payoff is given by

\[
E(b_{ij} \mid HH) = \frac{\alpha \pi^h}{\Delta \pi} (B(0) + c_{sim})
\] \hfill (10)

The lender’s feasibility condition, \( E[x_i \mid HH] \geq \rho + E[b_i \mid HH] \), gives us the set of projects that can be financed under simultaneous group lending.
\[ \bar{x} \geq \frac{\rho}{\pi h} + \frac{1}{\Delta \pi} B(\epsilon_{\text{sim}}) \]

### 4.2 Sequential Group Lending

If the lender allocates credit sequentially, only one borrower gets the loan from the lender in the first period. The remaining borrower in the group gets the loan only if the first borrower succeeds.

The lender randomly chooses a borrower in the group to lend to first. Let us call the first borrower \( B_1 \). Her peer \( B_2 \) can only borrow if \( B_1 \)'s project succeeds. As before, \( B_1 \) gets punished for the failure of her peer’s project. Additionally, with sequential group lending, \( B_2 \) is denied the opportunity to borrow if her peer’s project fails. The agents share the burden of failure equally as their payoffs are symmetric and the first period borrower is chosen randomly.

In sequential group lending, the borrowers alternate between the task of pursuing their project and monitoring their peer. When \( B_1 \) undertakes the project, she is monitored by \( B_2 \). Subsequently, their roles are reversed if \( B_1 \)'s project succeeds. The lender can distinguish between the following three states:

- \( f \) \( B_1 \)'s project fails
- \( sf \) \( B_1 \)'s project succeeds and \( B_2 \)'s project fails
- \( ss \) \( B_1 \) and \( B_2 \)'s projects succeed

The lender offers the borrowers a contract with outcome-contingent payoffs \((b_{ss}, b_{sf}, b_f)\). If \( B_1 \)'s project fails, both borrowers receive \( b_f \) and the game terminates. Conversely, if her project succeeds, \( B_2 \) gets the loan. If \( B_2 \)'s project succeeds (fails), both agents get a symmetrical payoff of \( b_{ss} \) (\( b_{sf} \)) and the game terminates. The timing of the game is as follows:
The lender offers $B_1$ and $B_2$ an identical contract $(b_{ss}, b_{sf}, b_f)$.

If they accept the contract, the game continues. Otherwise, it terminates.

t=1  $B_2$ chooses $c_2$, the intensity with which she monitors $B_1$.

t=2  $B_1$ chooses $e_1$, the effort level for her project.

t=3  $B_1$’s project outcome is realised.

  If $B_1$’s project fails, both agents get $b_f$. The game terminates.

  If $B_1$’s project succeeds, the game continues.

t=4  $B_1$ chooses $c_1$, the intensity with which she monitors $B_2$.

t=5  $B_2$ chooses $e_2$, the effort level for her project.

t=6  $B_2$’s project outcome is realised.

  If $B_2$’s project fails, both agents get $b_{sf}$. The game terminates.

  If $B_2$’s project succeeds, both agents get $b_{ss}$. The game terminates.

The limited liability constraint ensures that all payoffs are non-negative.

\[ b_{ij} \geq 0 \quad \forall \ i, j \]
\[ b_f \geq 0 \]

Again, the lender’s desired outcome is one where both borrowers choose to exert high effort on their respective projects. In Appendix 2, we show that the desired outcome is the SPNE of the game if the following condition is met.

\[ b_{ss} \geq \frac{1}{\pi h \Delta \pi} \max [B(c), c] \quad \text{(Condition 3)} \]

The condition states that the payoff should be high enough to induce the borrowers to monitor with intensity $c$ and exert high effort on their projects.
If condition 3 is satisfied, the game will have a SPNE where both borrowers will exert high effort for their respective projects.

**Proposition 3.** *Sequential group lending is feasible if the borrowers are allocated rents which satisfy condition 3.*

In sequential group lending, the lender only needs to satisfy the individual’s incentive compatibility condition associated with monitoring and effort. Unlike in simultaneous group lending, the lender does not have to satisfy the group’s collective incentive compatibility condition.

In simultaneous group lending, allowing the group to make the decisions on the tasks simultaneously meant that both tasks had to be incentivized simultaneously. By separating the decision temporally, the lender only has to incentivize the tasks individually at each stage.

For instance, by monitoring at \( t = 1 \), \( B_2 \) reduces the likelihood of getting a payoff of \( b_f = 0 \) if the game terminates prematurely. Similarly, by monitoring at \( t = 4 \), \( B_1 \) reduces the likelihood of \( B_2 \)’s project failing and her bearing the brunt of joint liability by receiving payoff \( b_{sf} = 0 \). Consequently, \( b_{ss} \) just has to compensates them both for resources expended in monitoring. Similarly, both borrowers would exert high effort if \( b_{ss} \) covered their opportunity cost of high effort. In the section below, we show that incentivizing the task individually is cheaper in terms of rents than incentivizing both tasks simultaneously.

The lender’s problem follows:

\[
\max_{b_{ij}} \ E [x \mid H] - E [b_{ij} \mid H]
\]

subject to Condition 3

To minimise the rents that the borrowers retain, the lender would like to
induce monitoring intensity $c_{seq}$ defined by

$$B(c_{seq}) = c_{seq}$$  \hspace{1cm} (11)$$

The borrower’s expected payoff is given by

$$E[b_{ij} \mid HH] = \frac{\pi h}{\Delta \pi} c_{seq}.$$  \hspace{1cm} (12)$$

Punishing the group if the first borrower’s project fails is expensive for the lender. The lender expects to pay the group more per unit capital lent in sequential as opposed to simultaneous group lending.\textsuperscript{11} Consequently, from the lender’s perspective, his capital is less productive in sequential group lending. Using the lender break-even condition, we find the set of all projects feasible under sequential group lending.

$$\bar{x} \geq \frac{\rho}{\pi h} + \frac{2}{(1 + \pi h)\Delta \pi} B(c_{seq})$$

### 4.3 Comparing Economic Rents

From Figure 1 it is clear that for all monitoring functions with the property

$B_c(c) < 0 \forall c \geq 0$, we would have $c_{sim} < c_{seq}$ and $B(c_{sim}) > B(c_{seq})$.\textsuperscript{12} That is, if monitoring reduces the borrower’s private benefits, the lender would always induce more monitoring in sequential as opposed to simultaneous group lending.

\textsuperscript{11}In simultaneous group lending, the lender lends 2 units of capital and expects to get an output of $2\pi h x$. In sequential group lending, the lender expects to lend $(1 + \pi h)$ units of capital and get an output valued at $\pi h(1 + \pi h)x$. He pays the borrowers $2b_{ss}$ with probability $\pi h^2$ in both cases. Consequently, the lender pays the borrower $b_{ss}$ and $\frac{2}{1 + \pi^2} b_{ss}$ per unit of capital lent in simultaneous and sequential group lending respectively.

\textsuperscript{12}Given that $AB$ and $OC$ intersect at $c = \frac{\pi h}{\Delta \pi} B(0)$ which is at a height greater than $B(0)$.
In both simultaneous and sequential group lending, effort gets incentivized along the segment ED in Figure 1. Incentivizing monitoring is more expensive in simultaneous group lending. This is because to incentivize monitoring, the lender has to satisfy the group’s collective incentive compatibility condition along the segment AB. In contrast, in sequential group lending, the tasks need to be incentivized individually and monitoring gets incentivized along segment OC.

Figure 1: Monitoring Intensities in Group Lending

High effort would be implemented in simultaneous and sequential group lending if the payoffs were above the segments EHB and EGC respectively. The lender’s problem gets solved at H in simultaneous and at G in sequential group lending. Consequently, the borrower’s payoff is higher in simultaneous as compared to sequential group lending. The corollary to proposition 3
follows.

**Corollary 1.** *In sequential group lending, the group’s collective incentive compatibility condition is slack.*

The group’s collective incentive compatibility condition gets satisfied at H in Figure 1. In sequential group lending, the lender offers the borrower a contract at G leaving the group’s collective incentive compatibility condition slack.

### 4.4 Collusion

Colluding does not require any side-contracting ability in simultaneous group lending. The borrowers take their monitoring and effort decisions simultaneously and consequently incur the cost of monitoring and obtain private benefits, simultaneously.

Conversely, colluding in sequential group lending is not trivial given that the decision on actions are separated temporally. The borrowers incur their monitoring costs and obtain private benefits at different points in time. Thus, to collude, they need to be able to sign and enforce contracts across time. For instance, by not monitoring, $B_1$ ($B_2$) saves on monitoring costs at $t = 4$ ($t = 1$) and $B_2$ ($B_1$) obtains the private benefits from low effort at $t = 5$ ($t = 2$). The subgame(s) of the sequential group lending game is (are) almost identical to the delegated monitoring case we analysed above.

In group lending, the group’s incentive compatibility condition (7) can also be interpreted as the no-collusion condition. Given that in simultaneous group lending, the borrowers do not need any ability to side contract to be able to coordinate on the no-monitoring low-effort equilibrium, the lender has to ensure that (7) is always satisfied. Otherwise, simultaneous group
lending is not feasible.

Conversely, as we know from corollary 1, the group’s collective incentive compatibility condition is slack in sequential group lending and the borrowers could potentially benefit from colluding, that is, by coordinating on the no-monitoring low-effort equilibrium.

Given that monitoring costs and private benefits are non-pecuniary, the borrowers would collude if either (a) the non-pecuniary costs and benefits were transferable amongst them or (b) if they had the ability to sign and enforce side contracts on actions across time. We summarise with the following proposition.

**Proposition 4.** The lender is able to exploit the group’s inability to fully side-contract on actions over time in sequential group lending to lower the borrower’s rents.

5 Group lending with varying efficiency of Peer Monitoring Technology

In this section we examine the effect of varying the efficiency of the peer monitoring technology. We introduce a parameter $\beta$ which measures the efficiency of the peer monitoring technology. Higher values of $\beta$ are associated with greater efficiency of the peer monitoring technology. We impose the following additional assumption on the monitoring function $B(c, \beta)$.

**Assumption 3** (Monitoring function $B(c, \beta)$).

1. $B(0, \beta) = B_0 > 0 \ \forall \beta \geq 0$

2. $B(c, \beta)$ is continuous and at least once differentiable $\forall \beta, c \geq 0$
iii. $B_c(c, \beta) < 0, \ B_\beta(c, \beta) < 0 \ \forall \ \beta, c \geq 0$

For any given $\beta$, $\bar{x}_{sim}$ and $\bar{x}_{seq}$, the least productive projects financed under simultaneous and sequential group lending respectively, are given by

$$\bar{x}_{sim} = \frac{\rho}{\pi^h} + \frac{1}{\Delta \pi} \left[ B(c_{sim}, \beta) \right]$$ (13)

$$\bar{x}_{seq} = \frac{\rho}{\pi^h} + \frac{2}{\Delta \pi} \left[ B(c_{seq}, \beta) \right]$$ (14)

where $c_{sim}$ and $c_{seq}$ are defined by (9) and (11) respectively.

**Proposition 5.** As the peer monitoring technology becomes more efficient, a greater range of projects is feasible under both group lending mechanisms.

![Figure 2: $c_{sim}$ and $c_{seq}$ as Monitoring Efficiency Varies](image)

We see the effects of a more efficient peer monitoring technology on the
borrower’s payoff in Figure 2. In Appendix D, we show that as the peer monitoring technology becomes more efficient ($\beta$ increases), the borrowers in both group lending mechanism get lower rents. $\bar{x}_{sim}$ and $\bar{x}_{seq}$ decrease as the lender is able to finance lower productivity projects.

**Proposition 6.** With an extremely efficient peer monitoring technology ($\beta \to \infty$), some socially viable projects are not feasible with simultaneous group lending, whereas all socially viable projects are feasible under sequential group lending.

In Appendix E, we show as $\beta \to \infty$, $\bar{x}_{sim} \to \frac{\nu}{\pi h} + \frac{\alpha B(0)}{\Delta r}$ and $\bar{x}_{seq} \to \frac{\nu}{\pi h}$. Consequently, $\bar{x} \in \left[ \frac{\nu}{\pi h}, \frac{\nu}{\pi h} + \frac{\alpha B(0)}{\Delta r} \right]$ is the set of socially viable projects that are not feasible in simultaneous group lending because of the rents allocated to the borrowers to satisfy the group’s collective incentive compatibility condition (7).

### 5.1 Linear Monitoring Function

Further, with a linear monitoring function of the form $B(c, \beta) = B(0) - \beta \cdot c$, we can find the conditions under which sequential group lending finances a greater range of projects than simultaneous group lending.

**Proposition 7.** With a linear monitoring function, we can show that if the peer monitoring technology is sufficiently efficient, a greater range of projects is financed with sequential as compared to simultaneous group lending.

In Appendix F, we show that if $\beta \geq -\frac{2}{\pi h} + \sqrt{(2 - \frac{h}{\pi})^2 + 4(k - 1)} > 0$, then $\bar{x}_{sim} \geq \bar{x}_{seq}$. (where $k = \frac{2}{1+\pi h}$). That is, with a sufficiently high $\beta$, a greater range of projects is feasible with sequential as compared to simultaneous group lending. This is because even though the borrower’s
rents are lower in sequential group lending, punishing the group when $B_1$’s project fails implies that the lender pays more per unit capital lent to the group, thus lowering his capital’s productivity. Thus, for sufficiently high values of $\beta$, the difference in the borrower’s rents overwhelms the difference in the productivity of the lender’s capital.

6 Conclusion

We compared the sequential lending mechanism with the simultaneous lending mechanism. With simultaneous group lending, the lender has to leave the borrowers sufficient rents to satisfy the group’s collective incentive compatibility condition. Given that the borrowers make their monitoring and effort choices simultaneously, the lender has to incentivize the group’s decisions on the two tasks, monitoring and effort, collectively.

Alternatively, the loans could be disbursed sequentially within the group with the proviso that the second borrower gets the loan only if the first borrower succeeds. With sequential group lending, the borrower’s effort and monitoring decisions are temporally separated. We show that in this case, the lender does not have to satisfy the group’s collective incentive compatibility condition. Thus, once the decisions are temporally separated, only the more expensive of the two tasks has to be incentivized.

Satisfying the group’s collective incentive compatibility condition requires that the lender leaves the borrowers higher rents in the simultaneous as opposed to sequential group lending. Thus, the advantage of lending sequentially in the group is that the lender has to allocate lower rents to the borrowers. Conversely, the disadvantage is that punishing the group for the first borrower’s project failure is expensive. It lowers the productivity of the
lender’s capital.

The difference in the borrower’s rents under the two group lending mechanism decreases as the peer monitoring technology becomes less efficient. We find that for a sufficiently efficient peer monitoring technology, a greater range of projects is feasible under sequential group lending. In this case, the difference in the borrowers’ rents overwhelms the difference in the productivity of capital in the two group lending mechanisms. Consequently, some socially viable projects that are infeasible under simultaneous group lending are feasible under sequential group lending. Conversely, if the peer monitoring technology is not sufficiently inefficient, a greater range of projects is feasible under simultaneous group lending.

Further, the borrower’s ability to collude through side contracting is irrelevant in simultaneous group lending. If the group’s collective incentive compatibility condition is satisfied, the group does not benefit from colluding. In sequential group lending, the group’s collective incentive compatibility condition remains slack. If the borrowers have an unlimited ability to side contract, they would benefit from colluding in this case. Consequently, the lender actually exploits the group’s inability to side contract across time to lower the rents left to the borrowers in sequential group lending.

A Individual Lending with Delegated Monitoring

The lender offers the borrower a contract \((b_s, b_f)\) and the monitor a contract \((w_s, w_f)\) which solves the following problem:

\[
\max_{b_i, w_s, c} E[x_i \mid H] - E[b_i \mid H] - E[w_i \mid H]
\]
subject to \[ E[b_i \mid H] \geq 0 \] (15)
\[ E[b_i \mid H] \geq E[b_i \mid L] + B(0) \] (16)
\[ b_i \geq 0; \quad i = s, f \] (17)
\[ E[w_i \mid H] - c \geq 0 \] (18)
\[ E[w_i \mid H] - c \geq E[w_i \mid L] \] (19)
\[ w_i \geq 0; \quad i = s, f \] (20)

where (15) and (18) are the participation constraints, (16) and (19), the incentive compatibility constraints and (17) and (20), the limited liability constraints of the borrower and the monitor respectively.

In the optimal contract, the borrower’s and monitor’s incentive compatibility constraints bind. Their respective participation constraints remain slack and their limited liability constraints bind only in the state \( f \). The lender offers the borrower and the monitor the following contracts:

\[ b_s = \frac{B(c_{dm})}{\Delta \pi}; \quad b_f = 0 \] (21)
\[ w_s = \frac{c_{dm}}{\Delta \pi}; \quad w_f = 0 \] (22)

where \( c_{dm} = B_{c}^{-1}(-1) \).
B Simultaneous Group Lending

For a subgame $\xi(c_1, c_2)$, $B_1$ and $B_2$’s respective payoffs from exerting effort $e_1$ and $e_2$ respectively are given by

$$\Pi_1[e_1, e_2, c_1, c_2] = E(b_{ij} | e_1, e_2) - c_1 + \left[ \frac{\pi^h - \pi_1}{\pi^h - \pi^l} \right] B(c_2)$$

$$\Pi_2[e_1, e_2, c_1, c_2] = E(b_{ij} | e_1, e_2) - c_2 + \left[ \frac{\pi^h - \pi_2}{\pi^h - \pi^l} \right] B(c_1)$$

where $\pi_k = \pi^h$ if $e_k = H$ and $\pi_k = \pi^l$ if $e_k = L$. For ease of exposition, we use $\bar{e}_1 \bar{e}_2(c_1, c_2)$ as a shorthand notation to refer to a particular outcome where $B_1$ and $B_1$ choose effort levels $e_1 = \bar{e}_1$ and $e_2 = \bar{e}_2$ respectively in the subgame $\xi(c_1, c_2)$. Thus, for instance, $LH(\bar{c}_1, \bar{c}_2)$ refers to a situation where $B_1$ and $B_2$ choose $c_1 = \bar{c}_2$ and $c_2 = \bar{c}_2$ at $t = 1$ and choose $e_1 = L$ and $e_2 = H$ at $t = 2$ respectively. Given our assumption of statistical independence of the projects, the likelihood of state $ss$, given the above effort levels, is $\pi^l \pi^h$.

Of the game described in Section 4.1, we analyse the subgames $\xi(c, c)$, $\xi(c, 0)$, $\xi(0, c)$ and $\xi(0, 0)$. In the subgame $\xi(c, c)$, $B_1$ does not deviate from $HH(c, c)$ if $\Pi_1[H, H, c, c] \geq \Pi_1[L, H, c, c]$, which gives us

$$b_{ss} \geq \frac{B(c)}{\pi^h \Delta \pi}. \quad \text{(Condition 1)}$$

$B_1$ does not deviate from $LL(c, c)$ if $\Pi_1[L, L, c, c] \geq \Pi_1[H, L, c, c]$, which gives us the condition

$$\frac{B(c)}{\pi^l \Delta \pi} \geq b_{ss}. \quad \text{(23)}$$

In subgame $\xi(c, c)$, $HH(c, c)$ and $LL(c, c)$ are Nash Equilibria if (Condition 1) and (23) satisfied. The borrowers would coordinate on $HH(c, c)$ if
\[ \Pi_1[H, H, c, c] \geq \Pi_1[L, L, c, c] \] giving us

\[ b_{ss} \geq \frac{B(c)}{\pi h^2 - \pi l^2}. \]  \hspace{1cm} (24)

In the subgame \( \xi(c, 0) \), \( B_1 \) does not deviate from \( HH(c, 0) \) if \( \Pi_1[H, H, c, 0] \geq \Pi_1[L, H, c, 0] \), giving us

\[ b_{ss} \geq \frac{B(0)}{\pi l \Delta \pi}. \]

\( B_1 \) does not deviate from \( LL(c, 0) \) if \( \Pi_1[L, L, c, 0] \geq \Pi_1[H, L, c, 0] \), giving us

\[ \frac{B(0)}{\pi l \Delta \pi} \geq b_{ss}. \]

\( B_2 \) does not deviate from \( HH(c, 0) \) if \( \Pi_2[H, H, c, 0] \geq \Pi_1[L, H, c, 0] \), which gives us

\[ b_{ss} \geq \frac{B(c)}{\pi h \Delta \pi}. \]

\( B_2 \) does not deviate from \( LL(c, 0) \) if \( \Pi_2[L, L, c, 0] \geq \Pi_1[L, L, c, 0] \), which gives us

\[ \frac{B(c)}{\pi l \Delta \pi} \geq b_{ss}. \]

Thus, in the subgame \( \xi(c, 0) \), \( LL(c, 0) \) is the only Nash Equilibrium if the following condition is met

\[ b_{ss} < \frac{B(0)}{\pi h \Delta \pi}. \]  \hspace{1cm} (25)
By symmetry, (25) would also ensure that \( LL(0, c) \) is the only Nash Equilibrium in the subgame \( \xi(0, c) \).

In the subgame \( \xi(0, 0) \), \( B_1 \) does not deviate from \( HH(0, 0) \) if \( \Pi_1[H, H, 0, 0] \geq \Pi_1[L, H, 0, 0] \), which gives us

\[
\begin{align*}
&b_{ss} \geq \frac{B(0)}{\pi h \Delta \pi}.
\end{align*}
\]

\( B_1 \) does not deviate from \( LL(0, 0) \) if \( \Pi_1[L, L, 0, 0] \geq \Pi_1[H, L, 0, 0] \), which gives us

\[
\begin{align*}
&\frac{B(0)}{\pi l \Delta \pi} \geq b_{ss}.
\end{align*}
\]

In the subgame \( \xi(0, 0) \), \( LL(0, 0) \) is the only Nash Equilibrium if (25) is satisfied. Moving up the game tree, \( c \) would be the best response to \( c \) if

\[
\Pi_1[H, H, c, c] \geq \max \left( \Pi_1[L, L, c, 0], \Pi_1[L, L, 0, c] \right)
\]

The condition given above would be satisfied if the following two conditions are satisfied.

\[
\begin{align*}
\Pi_1[H, H, c, c] &\geq \Pi_1[L, L, c, 0] \\
&b_{ss} \geq \frac{B(0)}{\pi h^2 - \pi l^2} \quad (26)
\end{align*}
\]

\[
\begin{align*}
\Pi_1[H, H, c, c] &\geq \Pi_1[L, L, 0, c] \\
&b_{ss} \geq \frac{B(c) + c}{\pi h^2 - \pi l^2} \quad (27)
\end{align*}
\]

This leaves us with \( HH(c, c) \) and \( LL(0, 0) \). The borrowers would prefer
\( HH(c, c) \) over \( LL(0, 0) \) if \( \Pi_1[H, H, c, c] \geq \Pi_1[L, L, 0, 0] \), which gives us the condition

\[
b_{ss} \geq \frac{B(0) + c}{\pi^2 - \pi'^2}.
\]

(Condition 2)

Condition 1, together with (23) and (25) give us a range for \( b_{ss} \). Condition 1 gives us the lower bound for the range. The upper bound of the range is given by either (23) or (25). Given that the lender’s objective is to minimise the borrower’s payoffs, he would ignore the upper bound. Further, if Condition 2 is satisfied, then (24), (26) and (27) would also be satisfied.

Consequently, if Condition 1 and Condition 2 are satisfied, the desired outcome is the SPNE of the game.

**C Sequential Group Lending**

\( B_1 \) and \( B_2 \)’s respective final payoffs are:

\[
\Pi_1[e_1, e_2, c_1, c_2] = \pi_1[\pi_2 b_{ss} + (1 - \pi_2)b_{sf}] - c_1 + (1 - \pi_1)b_f + \left[ \frac{\pi^h - \pi_1}{\pi^h - \pi_l} \right] B(c_2)
\]

\[
\Pi_2[e_1, e_2, c_1, c_2] = \pi_1[\pi_2 b_{ss} + (1 - \pi_2)b_{sf}] - c_2 + (1 - \pi_1)b_f + \left[ \frac{\pi^h - \pi_1}{\pi^h - \pi_l} \right] B(c_1)
\]

where \( \pi_k = \pi^h \) if \( e_k = H \) and \( \pi_k = \pi^l \) if \( e_k = L \). In the subgame \( \xi(c_2, e_1) \), \( B_2 \) chooses high effort level \( (e_2 = H) \) at \( t = 5 \) and \( B_1 \) chooses positive monitoring intensity \( (c_1 > 0) \) at \( t = 4 \) if the following conditions hold:

\[
\Pi_2[\pi_1, H, c_1, c_2] \geq \Pi_2[\pi_1, L, c_1, c_2]
\]

(28)

\[
\Pi_2[\pi_1, H, 0, c_2] \leq \Pi_2[\pi_1, L, 0, c_2]
\]

(29)

\[
\Pi_1[\pi_1, H, c_1, c_2] \geq \Pi_1[\pi_1, L, 0, c_2]
\]

(30)
If (28) and (30) are satisfied but (29) is not satisfied, $B_2$ would choose high effort at $t = 5$ in spite of $B_1$ choosing monitoring intensity $c_1 = 0$ at $t = 4$. Thus, it makes (29) irrelevant.

(28) and gives us the following condition:

$$b_{ss} - b_{sf} \geq \frac{B(c_1)}{\pi_1 \Delta \pi}$$  \hspace{1cm} (31)

(30) gives us

$$b_{ss} - b_{sf} \geq \frac{c_1}{\pi_1 \Delta \pi}.$$  \hspace{1cm} (32)

(31) and (32) can be summarised as:

$$b_{ss} - b_{sf} \geq \frac{1}{\pi_1 \Delta \pi} \max [B(c_1), c_1]$$

For the lender, $ss$ is the most informative state. Rewarding the agent in state $sf$, when $B_2$’s project fails is unnecessary. The lender can let the limited liability condition bind for $b_{sf}$ and set is to zero. The above condition can be restated as

$$b_{ss} \geq \frac{1}{\pi_1 \Delta \pi} \max [B(c_1), c_1].$$  \hspace{1cm} (33)

$B_1$ chooses high effort level ($e_1 = H$) at $t = 2$ and $B_2$ chooses positive monitoring intensity ($c_2 > 0$) at $t = 1$, if the following conditions hold:

$$\Pi_1[H, H, c_1, c_2] \geq \Pi_1[L, H, c_1, c_2]$$  \hspace{1cm} (34)

$$\Pi_1[H, H, c_1, 0] \leq \Pi_1[L, H, c_1, 0]$$  \hspace{1cm} (35)

$$\Pi_2[H, H, c_1, c_2] \geq \Pi_2[L, L, 0, 0]$$  \hspace{1cm} (36)
Again, if (34) and (36) are satisfied but (35) is not satisfied, $B_1$ would choose high effort at $t = 2$ in spite of $B_2$ choosing monitoring intensity $c_2 = 0$ at $t = 1$. Thus, it makes (35) irrelevant.

(34) gives us

$$\pi_2 b_{ss} + (1 - \pi_2)b_{sf} - b_f \geq B(c_2) \frac{\Delta\pi}{\Delta\pi}.$$  \hspace{1cm} (37)

(36) gives us

$$\pi_2 b_{ss} - (1 - \pi_2)b_{sf} - b_f \geq c_2 \frac{\Delta\pi}{\Delta\pi}.$$  \hspace{1cm} (38)

(37) and (38) give us

$$\pi^h b_{ss} + (1 - \pi^h)b_{sf} - b_f \geq \frac{1}{\Delta\pi} \max [B(c_2), c_2].$$

Again, $ss$ is the most informative state. Rewarding the borrowers in state $sf$ and $f$ is unnecessary. The lender can let the limited liability condition bind for $b_{sf}$ and $b_f$ and set them to zero. The above condition can be restated as

$$b_{ss} \geq \frac{1}{\pi^h \Delta\pi} \max [B(c_2), c_2].$$  \hspace{1cm} (39)

(33) and (39) give us

$$b_{ss} \geq \frac{1}{\pi^h \Delta\pi} \max [B(c), c].$$  \hspace{1cm} (Condition 3)

If this condition holds, it ensures that the game would have a SPNE in which both borrowers would monitor their respective peers with sufficient
intensity to ensure that both borrowers in turn exert high effort.

D Least Productive Project Financed with Varying Efficiency of Monitoring

$c_{sim}$ and $c_{seq}$ are defined by $B(c_{sim}, \beta) = \alpha(B(0) + c_{sim})$ and $B(c_{seq}, \beta) = c_{seq}$ respectively. From these conditions we can obtain that rate at which $c_{sim}$ and $c_{seq}$ change as $\beta$ changes.

\[
\frac{dc_{sim}}{d\beta} = \frac{B_\beta(c_{sim})}{\alpha - B_c(c_{sim})} \leq 0 \quad (40)
\]

\[
\frac{dc_{seq}}{d\beta} = \frac{B_\beta(c_{seq})}{1 - B_c(c_{seq})} \leq 0 \quad (41)
\]

We can find the rate at which $\bar{x}_{sim}$ and $\bar{x}_{seq}$ change by substituting $\frac{dc_{sim}}{d\beta}$ and $\frac{dc_{seq}}{d\beta}$ from the above expressions.

\[
\frac{d\bar{x}_{sim}}{d\beta} = \frac{\alpha}{\Delta \pi} \left[ \frac{B_\beta(c_{sim})}{\alpha - B_c(c_{sim})} \right] \leq 0 \quad (42)
\]

\[
\frac{d\bar{x}_{seq}}{d\beta} = \frac{2}{\Delta \pi (\pi^h + 1)} \left[ \frac{B_\beta(c_{seq})}{1 - B_c(c_{seq})} \right] \leq 0 \quad (43)
\]

As the peer monitoring technology becomes more efficient ($\beta$ increases), a greater range of projects is financed under both simultaneous and sequential group lending.
E  Group Lending with an Extremely Efficient Peer Monitoring Technology

The peer monitoring technology becomes extremely efficient as $\beta \to 0$.

$$\lim_{\beta \to \infty} c_{\text{sim}} = 0$$  \hspace{1cm} (44)

$$\lim_{\beta \to \infty} c_{\text{seq}} = 0$$  \hspace{1cm} (45)

With an extremely efficient peer monitoring technology, the lender induces negligible amounts of monitoring in the group members.

$$\lim_{\beta \to \infty} B(c_{\text{sim}}, \beta) = \alpha B(0)$$  \hspace{1cm} (46)

$$\lim_{\beta \to \infty} B(c_{\text{seq}}, \beta) = 0$$  \hspace{1cm} (47)

In sequential group lending, as $\beta \to \infty$, the borrower’s private benefits are driven down to almost 0 whereas in simultaneous group lending, they remain positive due to the scope for collusion amongst the borrowers.

$$\lim_{\beta \to \infty} \bar{x}_{\text{sim}} = \frac{\rho}{\pi^h} + \frac{\alpha B(0)}{\Delta \pi}$$  \hspace{1cm} (48)

$$\lim_{\beta \to \infty} \bar{x}_{\text{seq}} = \frac{\rho}{\pi^h}$$  \hspace{1cm} (49)

With borrowers retaining almost no rents in sequential group lending, all socially viable projects are feasible. In simultaneous group lending, due to the rents that borrowers retain, some projects namely $\bar{x} \in \left[\frac{\rho}{\pi^h}, \frac{\rho}{\pi^h} + \frac{\alpha B(0)}{\Delta \pi}\right]$ are not feasible even as $\beta \to \infty$. 

37
F  Linear Monitoring Function

With the linear monitoring function, \( B(c, \beta) = B(0) - \beta c \), we can find the values of \( c_{sim} \) and \( c_{seq} \).

\[
\begin{align*}
    c_{sim} &= \left[ \frac{1 - \alpha}{\beta + \alpha} \right] B(0) ;
    c_{seq} &= \left[ \frac{1}{1 + \beta} \right] B(0)
\end{align*}
\]

We look for conditions under which a greater range of projects is financed under sequential group lending.

\[
\bar{x}_{sim} \geq \bar{x}_{seq}
\]

\[
\alpha (B(0) + c_{sim}) \geq k \cdot c_{seq}
\quad \text{where} \quad k = \frac{2}{1 + \pi^h}
\]

Substituting the values of \( c_{sim} \) and \( c_{seq} \) gives us the following condition in terms of \( \beta \).

\[
\beta^2 + (2 - k) \beta - (k - 1) \geq 0
\]

Using the positive root of the quadratic equation, we find that the above condition is met when

\[
\beta \geq -\left(2 - \frac{k}{\alpha}\right) + \sqrt{\left(2 - \frac{k}{\alpha}\right)^2 + 4(k - 1)} > 0
\]

The right hand side is always positive since \( k > 1 \).

References


